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Power algorithms for several common tests.

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Monterey, California

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POWER ALGORITHMS FOR SEVERAL COMMON TESTS

by

Hur, Seong Pil

December 1986

Thesis Advisor: Donald R. Barr

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Power Algorithms for Several Common Tests

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ABSTRACT

Power algorithms for several common statistical tests are presented. In experimental design of operational tests and evaluations the selection of design parameters so as to attain an experiment with desired power is a difficult and important problem.

An interactive computer program is presented which uses the power algorithms for several tests and creates graphical presentations which can be used to assist decision makers in statistical design. Several common tests and associated parameters (such as sample size, types and levels of treatments, and alpha-level) are examined.

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I. INTRODUCTION

A. DESCRIPTION OF THE PROBLEM

A statistical design is a plan according to which an experiment is patterned. It provides the basis upon which appropriate statistical tests and inferences can be made after the experiment has been performed. The selection of the experimental design to be used in a given situation is extremely important because it plays a predominant role in the efficiency of the experiment, the precision with which the objectives are met, and the total effort (and cost) expended upon the experiment.

The concern of this thesis is the development of power algorithms for several common tests, including the F-test and t-test. The experimenter wants a test to achieve the correct decision with as high probability as possible in practical operational tests and evaluations given certain conditions hold.

B. SCOPE OF THE THESIS

An interactive computer program is developed which uses power algorithms for several common tests. The program can be used by decision makers who may not have deep knowledge of statistics or may be unfamiliar with the details of the design of experiments. The output helps the decision maker design experiments appropriate for testing hypotheses with common statistical tests. The experiment designer can use the program to evaluate the effect upon power of varying underlying design parameters such as sample size.

Because the underlying statistical noncentral distributions used in computing power do not have tables, approximation methods are used for computing the noncentral CDFs. Approximation methods were developed which are efficient, requiring reduced computer CPU time.

C. BACKGROUND

A **hypothesis** is a statement about the values of the parameters of a probability distribution. For example, suppose we think that the mean yield of a chemical process is more than 94.5 percent. Hypotheses to test this statement might be expressed formally as

$Ho : \mu \leq 94.5$

$Ha : \mu > 94.5$

The statement Ho : $\mu \leq 94.5$ is called the null hypothesis, and Ha : $\mu > 94.5$ is called the alternative hypothesis. The value of the mean specified in the null hypothesis might typically be determined in one of two ways. It may be the result of some theory or model regarding the process under study, or it may be the result of contractual specifications.

To test a hypothesis one usually devises a procedure for taking a random sample, computes an appropriate test statistic, and then rejects or declines to reject the null hypothesis Ho, depending on the outcome on the test statistic. Part of this procedure is the specification of the set of values for the test statistic which lead to rejection of Ho. This set of values is called the critical region or rejection region for the test.

Two kinds of errors may be committed when testing hypotheses. If the null hypothesis is rejected when it is true, then a type I error has occurred. If the null hypothesis is not rejected when it is false, then a type II error has been made. The probabilities of these two errors are given special symbols:

α = P(type I error) = P(reject Ho | Ho is true)

β = P(type II error) = P(fail to reject Ho | Ho is false)

Generally, the determination of β requires the use of a noncentral sampling distribution (e.g., noncentral F, noncentral χ^2 , and noncentral t) for which tables are not readily available. The noncentral sampling distribution depends on a parameter called the noncentrality parameter. The noncentrality parameter is usually a measure of the distance (in some sense) between the values of the parameter under the null and alternate hypotheses. Thus, type II error rates depend upon parameters of the distribution of the test statistic under the alternative hypothesis. The power of the test for a specified value of the hypothesized parameter is the probability the test would reject Ho when Ho is false, or $1 - \beta$.

In addition, whatever the test procedure is, rejecting Ho when Ho is not true is usually something we want a test to achieve with as high a probability as possible. Therefore we want the power of the test, for a given value of non-centrality parameter, to be high. Power algorithms are used to assist in the "best" selection of parameters for a statistical design. There do not exist simple closed form equations for computing the required probabilities from the common noncentral sampling distributions. Consequently, one must use numerical approximations to determine power. In this thesis we use various approximation methods to compute the cumulative probabilities for the noncentral F, noncentral χ^2, and noncentral t distributions.

II. NONCENTRAL CHI-SQUARE,T,F DISTRIBUTIONS

A. NONCENTRAL CHI-SQUARE DISTRIBUTION

If W1, W2, ..., Wn are independently distributed as $N(\mu_i, 1)$, i=1,2, ...,n, then $\sum Wi^2$ has a distribution known as a noncentral χ^2 (n, λ) where n is the number of degrees of freedom and $\lambda = \sum \mu_i^{\ 2}$ is the noncentrality parameter. When $\mu_1 = \mu_2 = \ldots = \mu_n = 0$, then $\lambda = 0$, and the noncentral χ^2 (n,0) reduces to the usual central χ^2 (n) with n degrees of freedom. The cumulative distribution function of χ^2 (n,λ) is,

$$F(x;n,\lambda) = P_r[\chi^2(n,\lambda) \leq x] \tag{2.1}$$

$$= e^{-\frac{1}{2}\lambda} \sum_{j=0}^{\infty} \frac{(\frac{1}{2}\lambda)^j}{j!} \frac{1}{2^{\frac{n}{2}+j}\,\Gamma(\frac{n}{2}+j)} \int_0^x y^{\frac{n}{2}+j-1} e^{-\frac{y}{2}} dy \ ; \ x > 0$$

while $F(x;n, \lambda) = 0$ for $x \leq 0$

It is possible to express $F(x;n,\lambda)$ for $x>0$, in an easily remembered form as a weighted sum of central χ^2 CDF's with weights equal to the probabilities of a Poisson distribution with expected value $\lambda/2$. That is,

$$F(x;n,\lambda) = \sum_{j=0}^{\infty} \left[\frac{(\frac{1}{2}\lambda)^j}{j!} e^{\frac{1}{2}\lambda} \right] \cdot P_r[\chi^2_{(n+2j)} \leq x] \tag{2.2}$$

Thus a χ^2 (n, λ) variable can be regarded as a mixture of central χ^2 variables. This interpretation is often useful in deriving the distribution of functions of noncentral χ^2 random variables.

The probability density function can, similarly, be expressed as a mixture of central χ^2 probability density functions

$$f(x) = c \sum_{j=0}^{\infty} \frac{\lambda^j x^{j-1}}{\Gamma(j+1)\, 2^j\, \Gamma(\frac{n}{2}+j)} \quad \text{where} \quad c = \frac{e^{-\lambda-\frac{x}{2}}\, x^{\frac{n}{2}}}{2^{\frac{n}{2}}} \tag{2.3}$$

The mean and variance of the distribution are

$$E(X) = n + 2\lambda \quad \text{and} \quad Var(X) = 2n + 8\lambda$$ [Ref. 1:p. 130].

B. NONCENTRAL F DISTRIBUTION

If Y_1 and Y_2 are independent and Y_1 is $\chi_1^{\,2}(n_1, \lambda)$ and Y_2 is $\chi^2(n_2)$ then

$$V = (Y_1 / n_1)/(Y_2 / n_2) \tag{2.4}$$

is distributed as $F'(n_1, n_2, \lambda)$, the noncentral F distribution with n_1 and n_2 degrees of freedom and noncentrality parameter λ. Its density function is given by

$$f(v) = c\sum_{k=0}^{\infty} \frac{\lambda^k n_1^k \Gamma\left(\frac{n_1}{2} + \frac{n_2}{2} + k\right) v^{k-1}}{\Gamma\left(\frac{n_1}{2} + k\right)(n_2 + n_1 v)^k} \; ; \; v \geq 0 \tag{2.5}$$

where

$$c = \frac{e^{-\lambda} n_1^{\frac{n_1}{2}} n_2^{\frac{n_2}{2}} v^{\frac{n_1}{2}}}{\Gamma\left(\frac{n_2}{2}\right)(n_2 + n_1 v)^{\frac{n_1}{2} + \frac{n_2}{2}}}$$

when $\lambda = 0$ this reduces to the density function of the central $F(n_1, n_2)$ distribution. The mean and variance of the distribution are

$$E(v) = \frac{n_2}{n_2 - 2}\left(1 + \frac{2\lambda}{n_1}\right) \qquad \text{for } n > 2$$

and

$$Var(V) = \frac{2n_2^2}{n_1^2(n_2 - 2)}\left[\frac{(n_1 + 2\lambda)^2}{(n_2 - 2)(n_2 - 4)} + \frac{n_1 + 4\lambda}{n_2 - 4}\right] \quad (\text{for } n_2 > 4)$$

When $\lambda = 0$, these reduce to the mean and variance of the central $F(n_1, n_2)$ distribution. Derivation of (2.5) is shown in [Ref. 1:p. 189].

The cumulative distribution of V can be expressed in terms of an infinite series of multiples of incomplete beta functions, as follows:

$$P_r[V \leq f_0] = \sum_{j=0}^{\infty}\left(\frac{[\frac{1}{2}\lambda_1]^j}{j!} e^{-\frac{1}{2}\lambda_1}\right) \cdot I_{\frac{n_1 f_0}{n_2 + n_1 f_0}}\left(\frac{1}{2}n_1 + j, \frac{n_2}{2}\right) \tag{2.6}$$

where $I_{\frac{n_1}{n_2+n_1 f_0}}\left(\frac{1}{2}n_1+j, \frac{n_2}{2}\right)$ is the incomplete beta function.

In this thesis, we use Paulson's Approximation from Severo and Zelen [Ref. 2]. Their approximation is as follows:

$$\Pr(V \le f_0) \sim \Phi(x) \tag{2.7}$$

where

$$x = \frac{\left(1-\frac{2}{9n_2}\right)\left(\frac{n_1 f_0}{n_1+\lambda_1}\right)^{\frac{1}{3}} - \left[1-\frac{2}{9}(n_1+2\lambda_1)(n_1+\lambda_1)^{-2}\right]}{\left[\frac{2}{9}(n_1+2\lambda_1)(n_1+\lambda_1)^{-2}+\frac{2}{9}n_2^{-1}\left(\frac{n_1 f_0}{n_1+\lambda_1}\right)^{\frac{2}{3}}\right]^{\frac{1}{2}}}, \text{ and}$$

Φ is the standard Normal CDF.

The following table shows some values obtained using this approximation, together with exact values of $\Pr(V \le fo)$. It can be seen in TABLE 1 that this approximation gives about 2 decimal place accuracy. Thus, this approximation provides adequate accuracy for computing the power of F-tests [Ref. 3:p. 202].

The program we used in the computation of noncentral F CDF values is shown in Appendix D.

C. NONCENTRAL T DISTRIBUTION

The ratio

$$T = (U+\lambda)/\sqrt{(Y/n)} \tag{2.8}$$

where U and Y are independent random variables distributed as standard normal (N(0,1)) and χ^2 with n degrees of freedom ,respectively, is said to have a noncentral t distribution with n degrees of freedom and noncentrality parameter λ. Sometimes λ^2 (or even $.5\lambda^2$), rather than λ, is termed the noncentrality parameter. If λ is equal to zero, the distribution is central t with n degrees of freedom.

The cumulative distribution of T is [Ref. 1:p. 201]

$$\Pr(T \le t) = \frac{1}{2^{\frac{1}{2}n-1}\Gamma(\frac{n}{2})} \int_0^\infty x^{n-1} e^{\frac{1}{2}x^2} \frac{1}{\sqrt{2\pi}} \int_0^{\frac{tx}{\sqrt{n}}} e^{[-\frac{1}{2}(u-\lambda)^2]} du\, dx. \qquad (2.9)$$

TABLE 1

CDF OF A NONCENTRAL F DISTRIBUTION

n1	n2	λ	fo	approx	exact
3	10	4	3.708	0.7499	0.745
		4	6.552	0.9192	0.918
		16	3.708	0.2023	0.206
		16	6.552	0.5190	0.517
3	20	4	3.098	0.7067	0.700
		4	4.938	0.8894	0.887
		16	3.098	0.1186	0.126
		16	4.938	0.3488	0.347
5	10	6	3.326	0.7337	0.731
		6	5.636	0.9143	0.914
		24	3.326	0.1553	0.158
		24	5.636	0.4629	0.461
5	20	6	2.711	0.6685	0.664
		6	4.103	0.8715	0.870
		24	2.711	0.0643	0.069
		24	4.103	0.2437	0.245
8	10	9	3.072	0.7159	0.714
		9	5.057	0.9087	0.908
		36	3.072	0.1166	0.119
		36	5.057	0.4088	0.408
8	30	9	2.266	0.581	0.578
		9	3.173	0.8157	0.813
		36	2.266	0.0146	0.017
		36	3.173	0.0846	0.088

In this thesis, we use an approximation method suggested by Abamowitz and Stegun [Ref. 4:p. 949].

$$\Pr(T \le t) \sim \Phi(x) \qquad (2.10)$$

where $x = \dfrac{t(1-\frac{1}{4n})-\lambda}{\left(1+\frac{t^2}{2n}\right)^{\frac{1}{2}}}$ and Φ is the standard Normal CDF.

The following table shows some values obtained using this approximation, together with exact values of Pr(T $\leq$ to). It can be seen in Table 2 that this approximation gives about 2 decimal place accuracy. Thus, this approximation provides adequate accuracy for computing the power of t-tests.

The program we used in the computation of noncentral t CDF values is shown Appendix E.

TABLE 2

CDF OF A NONCENTRAL T DISTRIBUTION

n	λ	to	approx	exact
4	0.112	4	0.017	0.01
	7.944	4	0.993	0.99
	2.24	10	0.026	0.01
	18.494	10	0.993	0.99
24	5.786	9.797	0.012	0.01
	13.981	9.797	0.991	0.99
	16.118	24.494	0.014	0.01
	33.078	24.494	0.992	0.99

III. POWER OF THE F-TEST

A. INTRODUCTION

Under appropriate conditions, the best test for testing equality of several means is the analysis of variance test. Analysis of variance has a wide application. It is one of the most useful techniques in the field of statistical inference. As in any hypothesis-testing situation, the power of the F test is of interest to the experimenter.

In this chapter we will discuss the power of F tests and provide an example. To give an overall evaluation of the power of F tests in the analysis of variance, we may use power curves. An important use of the power curve is to guide the experimenter in selecting the sample size (number of replicates) so that the design will be sufficiently sensitive to important potential differences in the treatments. We will consider power curves for one-way and multi-way analyses of variance (ANOVA's).

B. THE ONE-WAY CLASSIFICATION ANALYSIS OF VARIANCE

Suppose we have k different levels of a single factor that we wish to compare. The different levels of the factor are often called treatments. The observed responses from each of k treatments is a random sample on a random variable. The data would appear as in Figure 3.1.

		Observation			
Treatment	1	Y_{11}	Y_{12}	$\cdots$	Y_{1n}
	2	Y_{21}	Y_{22}	$\cdots$	Y_{2n}
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	k	Y_{k1}	Y_{k2}	$\cdots$	Y_{kn}

Figure 3.1 Typical Data for One-way Classification ANOVA.

We will find it useful to describe the observations by the linear statistical model:

$$Y_{ij} = \mu + \tau_i + \varepsilon_{ij} \tag{3.1}$$

where Y_{ij} is the $(ij)^{th}$ observation, μ is a parameter common to all treatments called the overall mean, τ_i is a parameter peculiar to the i^{th} treatment called the i^{th} treatment effect, and ε_{ij} is a random error component, assumed to be IID N $(0, \sigma^2)$.

The indices used are:

- i = the treatments, i = 1,2,...,k
- j = the replication per treatment, j = 1,2,...,n

The objective in the ANOVA is to test appropriate hypotheses about the treatment effects. The variance σ^2 is assumed constant for all levels of the factor. This model is called the one-way classification analysis of variance because only one factor is investigated.

We are interested in testing the equality of the k treatment effects, so the appropriate hypotheses are

$$\begin{aligned} Ho &: \tau_1 = \tau_2 = \ldots = \tau_k = \tau \\ Ha &: \quad \tau_i \neq \tau_j \text{ for some } i,j . \end{aligned}$$

That is, if the null hypothesis is true, then each observation is made up of the mean $\mu + \tau$ plus a realization of the random error ε_{ij}.

The results of the ANOVA procedure is summarized in Table 3.

We may use the expected values of the mean squares to verify that Fo (in Table 3) is an appropriate test statistic for Ho. From the expected mean square we see that, in general, MSE is an unbiased estimator of σ^2. Also, under the null hypothesis, MSt is an unbiased estimator of σ^2. However, if the null hypothesis is false, then the expected value of MSt is $\sigma^2 + (n \sum (\tau_i)^2)/(k-1)$. The expected value of the mean square error is σ^2.

Therefore, under the alternate hypothesis the expected value of the numerator of the test statistic (Fo) is greater than the expected value of the denominator and we would reject Ho with values of the test statistic which are too large. That is, we would reject Ho if

$$F_o > F_{\alpha,(k-1),(N-k)}$$

TABLE 3

THE ANALYSIS OF VARIANCE FOR A ONE-WAY CLASSIFICATION

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o
Between Treatments	SSt	k-1	MSt	MSt/MSE
Error(within treatments)	SSE	k(n-1)	MSE	
Total	SST	kn-1		

where the numerator and denominator df are k-1 and k(n-1)=N-k, and α is the type I error rate.

The power of the test is:

$$1-\beta = P(F_o > F_{\alpha,(k-1),(N-k)} \mid \text{Ho is false}) \qquad (3.2)$$

To evaluate the β in Equation 3.2 we need to know the distribution of the test statistic F_o if the null hypothesis is false. It can be shown that, if Ho is false, the statistic F_o has the noncentral F distribution with k-1 and k(n-1) degrees of freedom and noncentrality parameter λ, given by

$$\lambda = \frac{n \sum (\tau_i)^2}{\sigma^2}$$

The noncentrality can be interpreted as the squared standardized distance between the origin and $(\tau_1, \tau_2, \ldots, \tau_k)$. The ratio $\sum (\tau_i)^2 / \sigma^2$ is called the squared standardized distance. If only an estimate of σ^2 is available, one may replace σ^2 with the estimate [Ref. 5:p. 34].

C. THE MULTI-WAY CLASSIFICATION ANALYSIS OF VARIANCE

Many experiments require a study of the effects of two or more factors. It can be shown that, under certain conditions, factorial arrangements are the most efficient designs for this type of analysis.

One of the simplest factorial experiments involves only two factors or sets of treatments say factor A and factor B. Suppose there are a levels of factors A and b levels of factor B, and these are arranged in a 2-way factorial design; that is, each replication of the experiment contains all ab treatment combinations. Assume there are n replications of the experiment, and let Y_{ijk} represent the observation taken under the i^{th} level of factor A and the j^{th} level of factor B in the k^{th} replication.

The data can be summarized as shown in Figure 3.2. The order in which the abn observations are taken is selected at random.

Factor B

		1	2	...	b
	1	$Y_{111}, Y_{112}, \ldots, Y_{12n}$	$Y_{121}, Y_{122}, \ldots, Y_{12n}$	...	$Y_{1b1}, Y_{1b2}, \ldots, Y_{1bn}$
	2	$Y_{211}, Y_{212}, \ldots, Y_{21n}$	$Y_{221}, Y_{222}, \ldots, Y_{22n}$	...	$Y_{2b1}, Y_{2b2}, \ldots, Y_{2bn}$
Factor A	. . .	. . .	. . .	. . .	. . .
	a	$Y_{a11}, Y_{a12}, \ldots, Y_{a2n}$	$Y_{a21}, Y_{a22}, \ldots, Y_{a2n}$	...	$Y_{ab1}, Y_{ab2}, \ldots, Y_{abn}$

Figure 3.2 Typical Data notation for a Two-way Classification..

The observations may be described by the linear model:

$$Y_{ijk} = \mu + \tau_i + \beta_j + \tau\beta_{ij} + \varepsilon_{ijk}, \tag{3.3}$$

where

- μ = the overall mean effect;
- τ_i = the true effect of i th level of factor A, i = 1,2,...,a;
- β_j = the true effect of j th level of factor B, j = 1,2,...,b;
- $\tau\beta_{ij}$ = the effect of the interaction between τ_i and β_j ; and
- ε_{ijk} = a random error component, assumed to be IID N $(0,\sigma^2)$.

Both factors are assumed to be fixed. It is usually assumed that the treatment effects are defined as deviations from the overall mean, so $\sum \tau_i = 0$ and $\sum \beta_j = 0$. Similarly, the interaction effects are fixed and usally defined so that $\sum (\tau\beta)_{ij} = 0$, where the summation is over either i or j. Since there are n replicates of the experiment, there are a total of abn observations.

We are interested in testing various hypothesis about the parameters in equation 3.3 . An appropriate hypothesis testing procedure would again be analysis of variance. More specifically, as we are considering two controllable sources of variation (A and B), the procedure is called the two-way classification analysis variance.

In order to test the hypothesis Ho ; $\tau_i = 0$ for i=1,2,...,a (no row factor effects), Ho ; $\beta_j = 0$ for j=1,2,...,b (no column factor effects), and Ho ; $(\tau\beta)_{ij} = 0$ (no interaction effects), we can express the total sum of square as:

$$SST = SSA + SSB + SSAB + SSE. \tag{3.4}$$

Here, SSA is a sum of squares due to "rows" or factor A, SSB is a sum of squares due to "columns" or factor B, SSAB is a sum of squares due to the interaction between A and B, and SSE is a sum of squares due to error. The degrees of freedom associated with each sum of squares are shown in Figure 3.3.

If we assume ε_{ijk} are IID $N(0,\sigma^2)$ and apply Cochran's theorem (Theorem 3-1) under the null hypothesis of no effects, each sum of squares on the right-hand side of Table 4 when divided by σ^2 is distributed as χ^2 with the indicated number of degrees of freedom, and these statistics are independent.

Theorem 3-1 (COCHRAN). Let Z_i be IID N(0,1) for i=1,2,...,ν and suppose $\sum Z_i^2 = Q_1 + Q_2 + Q_3 + \ldots + Q_s$ where s $\leq$ ν, and the quadratic form Q_i has ν_i degrees of freedom (i=1,2,...,s). Then $Q_1, Q_2, \ldots, Q_s$ are independent chi-square random variable with $\nu_1, \nu_2, \ldots, \nu_s$ degrees of freedom, respectively, if and only if

$$\nu = \nu_1 + \nu_2 + \ldots + \nu_s.$$

Effect	Degrees of freedom
A	a-1
B	b-1
AB interaction	(a-1)(b-1)
Error	ab(n-1)
Total	abn-1

Figure 3.3 Table of degrees of freedom with sums of squares..

Assuming that factors A and B are fixed, the expected values of the mean squares are:

$$E(MSA) = \sigma^2 + (bn \sum \tau_i^{\,2})/(a-1) \; ; \tag{3.5}$$

$$E(MSB) = \sigma^2 + (bn \sum \beta_i^{\,2})/(b-1) \; ; \tag{3.6}$$

$$E(MSAB) = \sigma^2 + (n \sum \sum (\tau \beta)_{ij}^{\,2})/ (a-1)(b-1) \; ; \text{ and} \tag{3.7}$$

$$E(MSE) = \sigma^2 . \tag{3.8}$$

Therefore, to test the hypothesis Ho : $\tau_i = 0$; i=1,2,...,a (no row factor effects), and Ho ; $\beta_j = 0$; j=1,2,...,b (no column factor effects), and Ho : $(\tau \beta)_{ij} = 0$ (no interaction effects), we would divide the corresponding mean square by the mean square error. Under the null hypothesis of no effect, this ratio will follow an F distribution with appropriate numerator degrees of freedom and ab(n-1) denominator degrees of freedom, and the critical region will be located in the upper tail. The test procedure is usually summarized in an analysis of variance table, such as shown in Table 4.

TABLE 4

THE ANALYSIS OF VARIANCE FOR A TWO-WAY CLASSIFICATION

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	SSA	a-1	MSA	MSA/MSE
B treatments	SSB	b-1	MSB	MSB/MSE
Interaction	SSAB	(a-1)(b-1)	MSAB	MSAB/MSE
Error	SSE	ab(n-1)	MSE	
Total	SST	abn-1		

To the compute the power of tests in two-way ANOVAs. the procedure is the same as in the one-way case. Relationships among λ, the numerator degrees of freedom and the denominator degrees of freedom are shown in Table 5 .

In a similar way one can expand to the general multi-way ANOVA procedure [Ref. 5:p. 124].

D. THE ALGORITHMS AND FLOWCHART

A program for computing a power table and power curve in general ANOVA's (fixed model) is shown in Appendix F. The power of one-way ANOVA's are solved with these programs, using the following the sequence:

1) Determine what variable to include, such as sample size vs power, number of treatments vs power, α-level vs power, or noncentrality vs power.
2) Determine the values of the maximum, minimum, and increment of the variable chosen.
3) Given input data, compute the critical value, using the inverse of the central F-distribution (Appendix A).
4) Compute the power value in each case; that is, the CDF of the noncentral F-distribution (Chapter II).
5) Print the power table or the power curve.

TABLE 5

NONCENTRALITY PARAMETERS FOR POWER IN A TWO-WAY ANOVA

Factor	λ	Numerator DOF	Denominator DOF
A	$\frac{bn\sum\tau_i^2}{\sigma^2}$	a-1	ab(n-1)
B	$\frac{an\sum\beta_j^2}{\sigma^2}$	b-1	ab(n-1)
AB	$\frac{n\sum\sum\tau\beta_{ij}^2}{\sigma^2}$	(a-1)(b-1)	ab(n-1)

Multi-way ANOVAs have the same algorithms but multi-way ANOVA may include tests of interaction effects. The program considers only up to three-way interaction effects. We can explain how to compute degrees of freedom of error term in m-way ANOVA's, assuming the balanced case, as follows: Total degrees of freedom is DOF(Total) = n $\sum$ k(i) -1, where k(i) is the number of levels of the i^{th} factor [Ref. 9].

(1) If the model has only main effects without any interaction effects,

DOF1(Error) = DOF(Total) - $\sum$ (k(i)-1).

(2) If the model has several factors and only 2-way interaction effects,

DOF2(Error) = DOF1(Error) - $\sum\sum$ (k(i)-1)(k(j)-1).

(3) IF the model has several factors and only 3-way interaction effects,

DOF3(Error) = DOF1(Error) - $\sum\sum\sum$ (k(i)-1)(k(j)-1)(k(k)-1).

(4) If the model has several factors and 2-way and 3-way interaction effects,

DOF4(Error) = DOF3(Error) - $\sum\sum$ (k(i)-1)(k(j)-1).

If the model has more than 3-way interaction effects, then the user must modify the degrees of freedom of the error terms accordingly. A flowchart is shown in Figure 3.4.

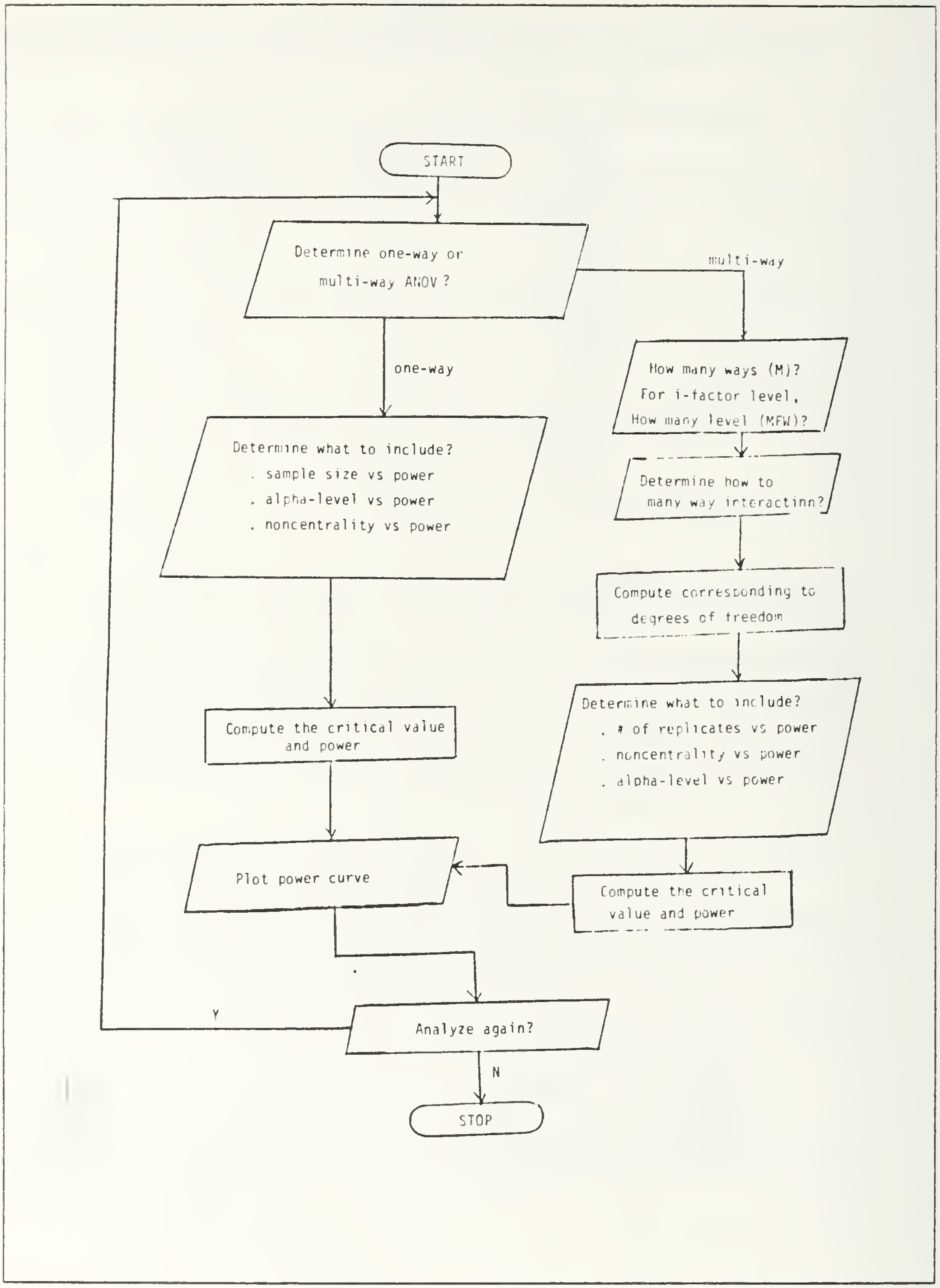


Figure 3.4 System flowchart for F-test power.

E. EXAMPLES OF F-TESTS

1. Example of One-way ANOVA test (number of replicates vs power)

a. Scenario

A manufacturer suspects that the batches of raw material furnished by his supplier differ significantly in calcium content. There are a large number of batches currently in the warehouse. Five of those are randomly selected for study. A chemist wants to know the appropriate sample size per each batch, in order to test

$$Ho : \tau_1 = \tau_2 = \ldots = \tau_k$$

against

$$Ha : \quad \tau_i \neq \tau_j \text{ for some } i,j$$

The chemist would like to know how many replicates to run if it is important to reject Ho with probability at least 0.9 when the standardized distance square ($\sum \tau_i^2 / \sigma^2$) is 2 and $\alpha = .05$. Thus he would like to know what the power of the F-test is for a range of possible replicates. He decides to check replicates from 2 to 12 in increments of 1.

b. Inputs

1) Select one-way ANOVA test (the number of replicates vs power).
2) The number of treatment = 5.
3) α -level = .05
4) Standardized distance square = 2.
5) Maximum replicates is 12, minimum replicates is 2, increment is 1.

c. Start program

Do you want to analyze one way ANOVA(y/n)?

y

Do you want to plot n (# of observation per treatment vs power)(y/n)?

y

Number of k(# of treatment)?

?

5

Alpha- level?

?

.05

Standardized squared distance value ?

?

2.

Maximum n value (the maximum value on X-axis)?

?

12

Minimum n value (the minimum value on X-axis)?

(condition :N must be more than 2)

?

2

Increment n value ?

?

1

d. Output

The output is presented in Table 6

TABLE 6

OUTPUT OF THE ONE-WAY ANOVA EXAMPLE (REPLICATES VS POWER)

DOF1	DOF2	α	F-INVERSE	λ	POWER
4	5	0.05	5.29087	4.00	0.15373
4	10	0.05	3.52496	6.00	0.30024
4	15	0.05	3.10397	8.00	0.44935
4	20	0.05	2.91676	10.00	0.58677
4	25	0.05	2.81108	12.00	0.70332
4	30	0.05	2.74323	14.00	0.79562
4	35	0.05	2.69600	16.00	0.86449
4	40	0.05	2.66122	18.00	0.91328
4	45	0.05	2.63455	20.00	0.94631
4	50	0.05	2.61345	22.00	0.96776
4	55	0.05	2.59634	24.00	0.98118

# of replicate		POWER	
NN=	2.	POWER=	0.15373
NN=	3.	POWER=	0.30024
NN=	4.	POWER=	0.44935
NN=	5.	POWER=	0.58677
NN=	6.	POWER=	0.70332

NN= 7. POWER= 0.79562

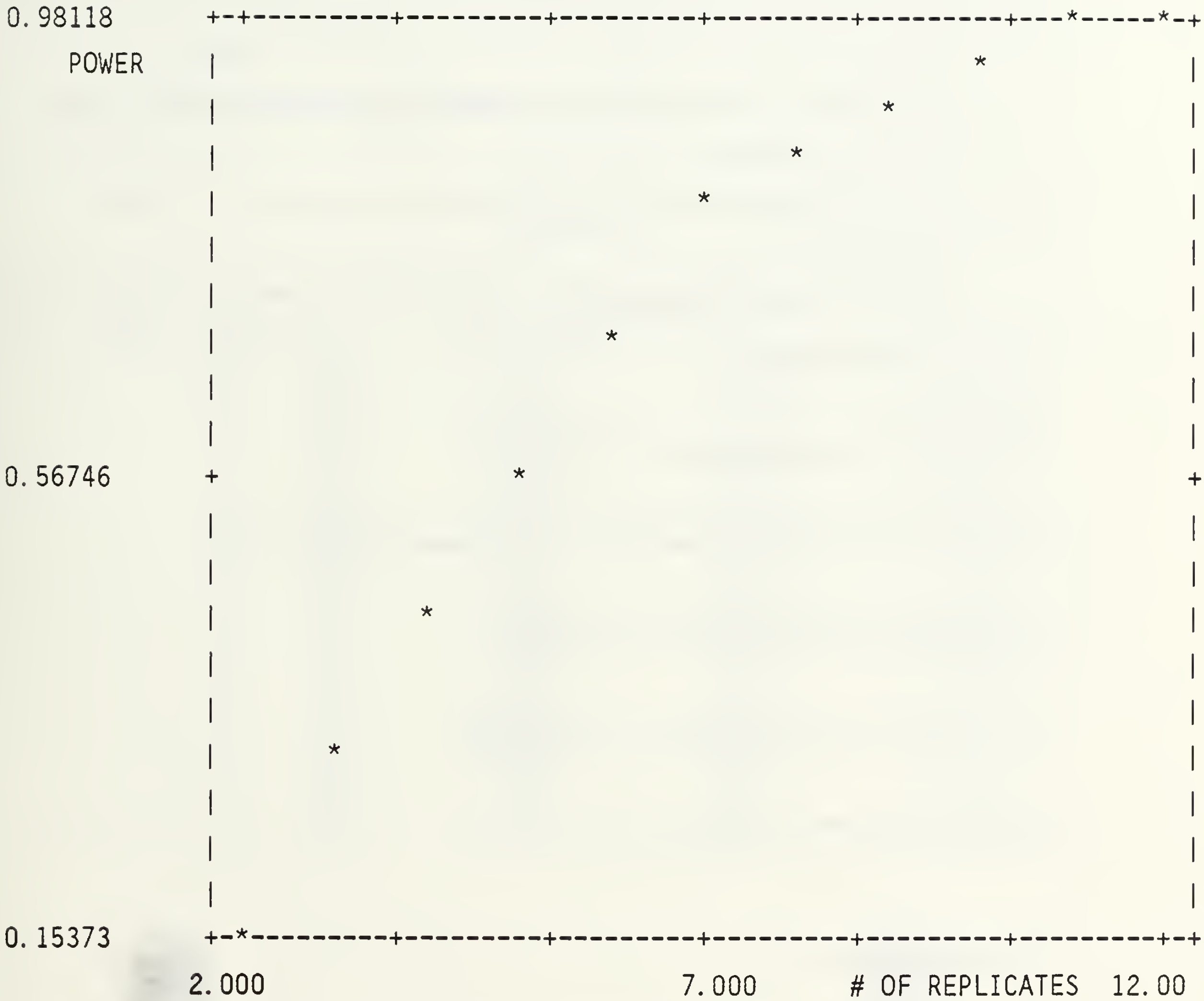
NN= 8. POWER= 0.86449

NN= 9. POWER= 0.91328

NN= 10. POWER= 0.94631

NN= 11. POWER= 0.96776

NN= 12. POWER= 0.98118



X-SCALE: "-"= 0.125E+00 UNITS

Y-SCALE: "|"= 0.138E-01 UNITS

2. **Example of one-way ANOVA test (noncentrality vs power)**

a. Scenario

Five brands of batteries are under study. It is suspected that the life (in weeks) of the five brands is different. Five batteries of each brand are tested. A manufacturer wants to know the power as a function of the standardized squared distances. That is, would like to know what the power of the F-test is for a range of possible standardized distances square. He decides to check standardized squared distances from 1 to 5 in .2 increment, where n=5 and α = .05.

b. Inputs

1) Select one-way ANOVA test (the standardized distance square vs power).
2) Number of treatment = 5.
3) Number of replicates = 5.
4) α -level = .05.
5) Maximum standardized distance square is 5 and minimum standardized distance square is 1 and the increment is .2.

c. Start program

Start program:

Do you want to analyze one-way ANOVA(y/n)?

y

Do you want to plot n(# of observation per treatment) vs power(y/n)?

n

Do you want to plot alpha-level vs power(y/n)?

n

Do you want to plot noncentrality vs power(y/n)?

y

# of observations per treatment (n=) ?

?

5

# of treatment (k=)?

?

5

Alpha-level ?

?

.05

Maximum standardized squared distance value range ?

?

5

Minimum standardized squared distance value range ?

?

1.

Increment standardized squared distance value range ?

?

.2

d. Output

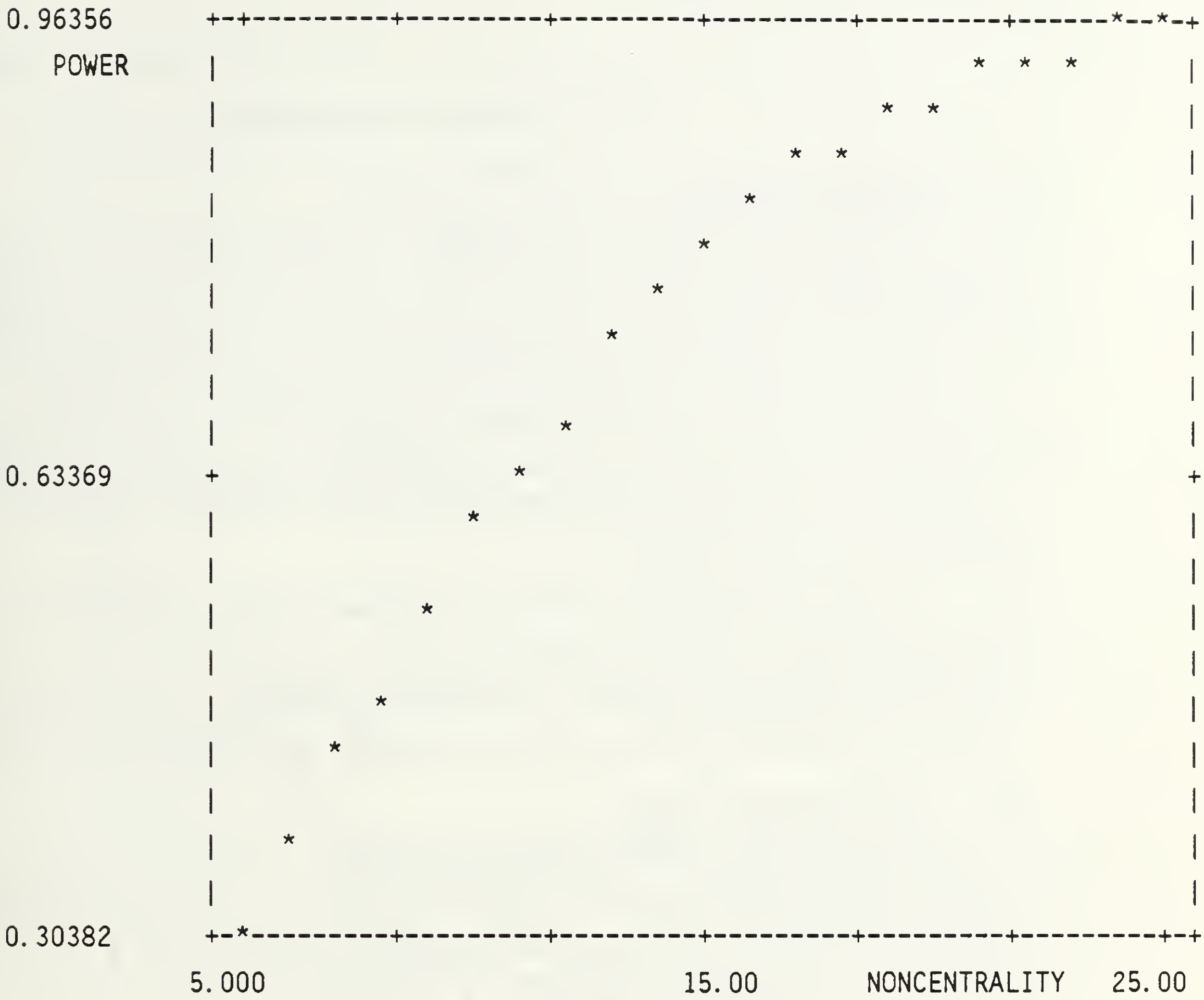
The screen output (Table 7) is as follows:

TABLE 7

OUTPUT OF THE ONE-WAY ANOVA EXAMPLE (NONCENTRALITY VS POWER)

DOF1	DOF2	α	F-INVERSE	λ	POWER
4	20	0.05	2.91676	5.00	0.30382
4	20	0.05	2.91676	6.00	0.36314
4	20	0.05	2.91676	7.00	0.42203
4	20	0.05	2.91676	8.00	0.47946
4	20	0.05	2.91676	9.00	0.53461
4	20	0.05	2.91676	10.00	0.58677
4	20	0.05	2.91676	11.00	0.63550
4	20	0.05	2.91676	12.00	0.68050
4	20	0.05	2.91676	13.00	0.72163
4	20	0.05	2.91676	14.00	0.75885
4	20	0.05	2.91676	15.00	0.79223
4	20	0.05	2.91676	16.00	0.82192
4	20	0.05	2.91676	17.00	0.84812
4	20	0.05	2.91676	18.00	0.87108
4	20	0.05	2.91676	19.00	0.89107
4	20	0.05	2.91676	20.00	0.90835
4	20	0.05	2.91676	21.00	0.92321
4	20	0.05	2.91676	22.00	0.93591
4	20	0.05	2.91676	23.00	0.94672
4	20	0.05	2.91676	24.00	0.95586
4	20	0.05	2.91676	25.00	0.96356

NONCENTRALITY		POWER	
NN=	5.0	POWER=	0.30382
NN=	6.0	POWER=	0.36314
NN=	7.0	POWER=	0.42203
NN=	8.0	POWER=	0.47946
NN=	9.0	POWER=	0.53461
NN=	10.0	POWER=	0.58677
NN=	11.0	POWER=	0.63550
NN=	12.0	POWER=	0.68050
NN=	13.0	POWER=	0.72163
NN=	14.0	POWER=	0.75885
NN=	15.0	POWER=	0.79223
NN=	16.0	POWER=	0.82192
NN=	17.0	POWER=	0.84812
NN=	18.0	POWER=	0.87108
NN=	19.0	POWER=	0.89107
NN=	20.0	POWER=	0.90835
NN=	21.0	POWER=	0.92321
NN=	22.0	POWER=	0.93591
NN=	23.0	POWER=	0.94672
NN=	24.0	POWER=	0.95586
NN=	25.0	POWER=	0.96356



X-SCALE: "-" = 0.250E+00 UNITS

Y-SCALE: "|" = 0.110E-01 UNITS

IV. POWER OF THE T-TEST

A. INTRODUCTION

In testing the hypothesis that the mean of a normal distribution is equal to μ_0 when the standard deviation is unknown, the t statistic may be used. This test-statistic is a function of the sample mean, the sample standard deviation, and the sample size.

The t-test has a very wide application in research areas. For example, we may wish to know whether product A is better than product B; or whether the outputs of machines C and D form a homogeneous mass of product; and so forth. To answer such questions we can employ the t-test which is one of the most useful techniques in the field of statistical inference.

In such hypothesis-testing situations the power of t-tests is usually of interest to the experimenter. We may use the power curve to evaluate the power of the t-test. The required sample size might be determined by referring to a set of power curves. We will consider the power curves for the one-sample and two-sample cases. [Ref. 10]

B. ONE-SAMPLE T-TEST

We assume that a random sample, X1, X2, X3, ..., Xn of size n is taken from a normal population with mean μ and standard deviation σ. Suppose we wish to test the hypothesis that $\mu \leq \mu_0$ against the alternative hypothesis that $\mu > \mu_0$. The procedure is to reject the hypothesis Ho : $\mu \leq \mu_0$ if $((\overline{X} - \mu_0) \sqrt{n}) / S > t_\alpha$, and to accept Ho if otherwise where $\overline{X}$ and S are the sample mean and sample standard deviation and where t_α is the $1 - \alpha^{th}$ percentage point of the Student t-distribution with n-1 degrees of freedom, i.e.,

$$P(\text{ Student t} > t_\alpha) = \alpha \tag{4.1}$$

With critical region (t_α , ∞), the hypothesis $\mu = \mu_0$ is rejected with probability α when Ho is true. If the mean of the normal distribution is $\mu_1 > \mu_0$, then the power of the above test is

$$P(\text{ noncentral t} > t_\alpha \mid \delta = ((\mu_1 - \mu_0) \sqrt{n})/ \sigma) = 1- \beta \tag{4.2}$$

where δ is the noncentrality parameter for the noncentral t-distribution with n-1 degrees of freedom [Ref. 6:p .320].

C. TWO-SAMPLE T-TEST

Suppose we are given two random samples from two normal populations X1, X2, ..., Xn and Y1, Y2, ..., Ym, where the X's and Y's are independent with mean values $\mu_x = \mu_y$ and common unknown variances $\sigma_x{}^2 = \sigma_y{}^2 = \sigma^2$. Suppose we wish to test the null hypothesis Ho : $\mu_x \leq \mu_y$ against the alternative hypothesis Ha that $\mu_x > \mu_y$. The procedure is to reject the hypothesis Ho if $(\overline{X} - \overline{Y}) / \surd (S_p{}^2 (1/n + 1/m)) > t_\alpha$ where t_α is a critical value of the Student t-distribution based on $m+n-2$ degrees of freedom. In this case take $\delta = (\mu_x - \mu_y) / \sigma \surd (1/n + 1/m)$ so the probability of rejecting the hypothesis when $\mu_x > \mu_y$ is equal to

$$P(\text{ noncentral } t > t_\alpha \mid \delta) = 1- \beta \tag{4.3}$$

where δ is the noncentrality parameter for the noncentral t-distribution with m+n-2 degrees of freedom [Ref. 6:p .321].

D. TWO-SIDED T-TEST

For tests where the alternative hypothesis specifies that $\mu \neq \mu_0$ in the test of section B or $\mu_x \neq \mu_y$ in the test of section C, where the direction can be either up or down, i.e, $\mu > \mu_0$ or $\mu < \mu_0$ and $\mu_x > \mu_y$ or $\mu_x < \mu_y$, we have what is known as a two-sided test. In this case the rejection procedure for the test of section B specifies that Ho is rejected if either $((\overline{X} - \mu_0) \surd n) / S > t_{\alpha/2}$ or $((\overline{X} - \mu_0) \surd n) / S < - t_{\alpha/2}$ and the power of the test is

$$P(\text{ noncentral } t > t_{\alpha/2} \mid \delta) + P(\text{ noncentral } t < -t_{\alpha/2} \mid \delta) = 1 - \beta \tag{4.4}$$

where $\delta = (\mu_1 - \mu_0) \surd n / \sigma$ [Ref. 6:p .323].

E. THE ALGORITHMS AND FLOWCHART

Listings of programs for generating the power table and curve for t-tests are given in the Appendix E.

The following steps are used to compute the power:

1) Determining whether one-sample or two-sample t-tests should be used.

2) After determining the test, give the input data. (n, α, μ_0, σ)

3) Determine whether the one-sided or two-sided test should be used.

4) Compute the critical region (inverse t-values). (The computation method is provided in Appendix B)

5) Compute the power corresponding to the chosen user option. (The computation method is discussed in Chapter II)

6) Plot the power corresponding to each above option.

A flowchart is shown in Figure 4.1a

F. EXAMPLES OF T-TESTS

1. Example of one sample one-sided t-test

a. Scenario

The supervisor in an electronic instrument plant is concerned with the repair time of an electronic equipment. Standards specify that the mean repair time must be at most 40 hours. For power computation, it is assumed that the variance of repairing time is 4.0. The appropriate hypothesis are

$$Ho : \mu \geq 40,$$
$$Ha : \mu < 40.$$

The experimenter decides to use a sample of n = 25 observations and α = .05. He would like to know what the power of the t-test is for a range of possible population means. He decides to check means from 38 to 40 in .1 increments.

b. Inputs

INPUTS:

step 1: one sample one-sided t-test

step 2: $\mu_0 = 40$

step 3: $\sigma = 2.0$

step 4: $\alpha = .05$

step 5: sample size = 25

step 6: maximum μ_1 = 40., minimum μ_1 = 38, increment = .1

c. Start program

START PROGRAM:

Do you want to test one sample t-test(y/n)?

y

Sample size ?

?

25

Alpha-level ?

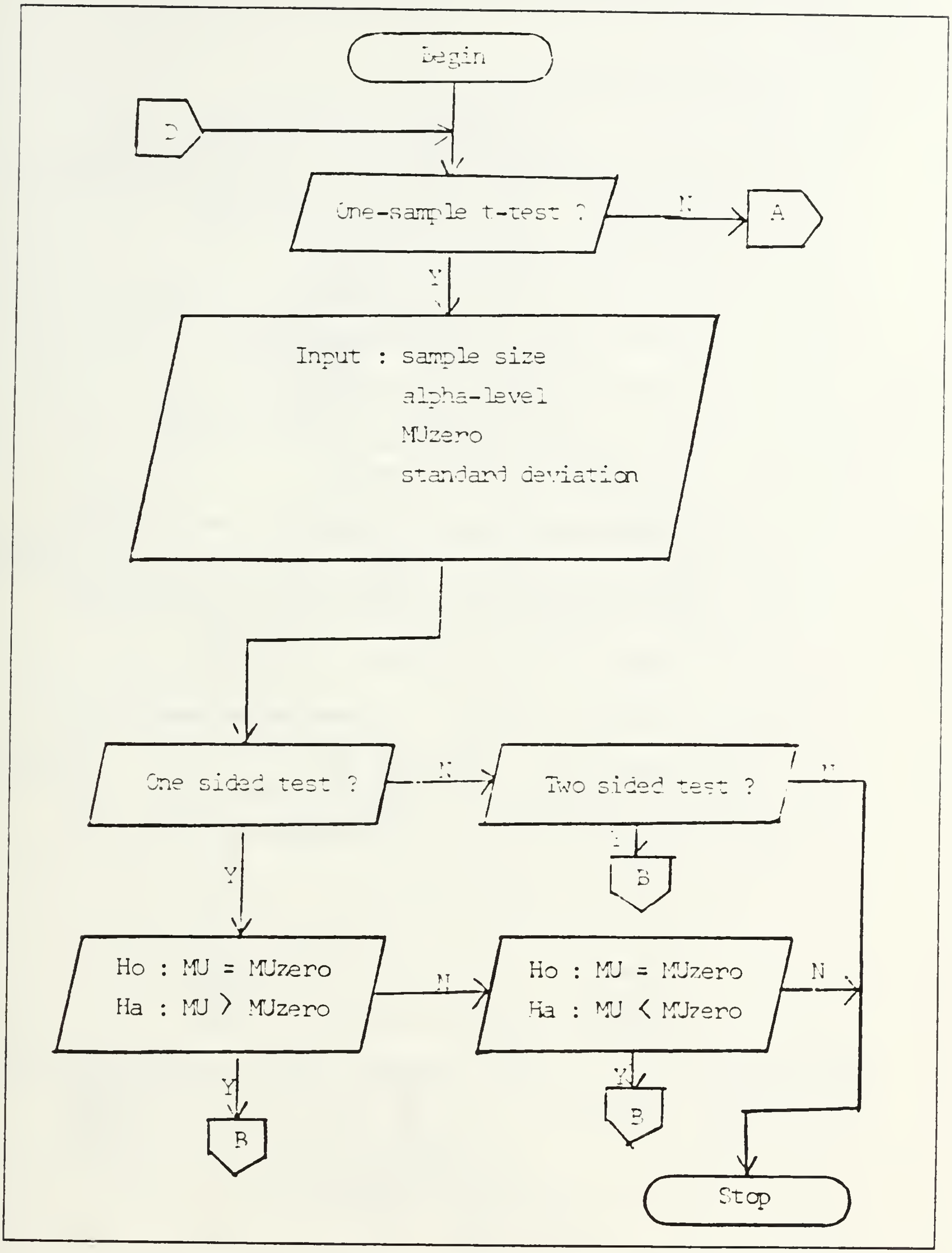


Figure 4.1a System flowchart for t-test power.

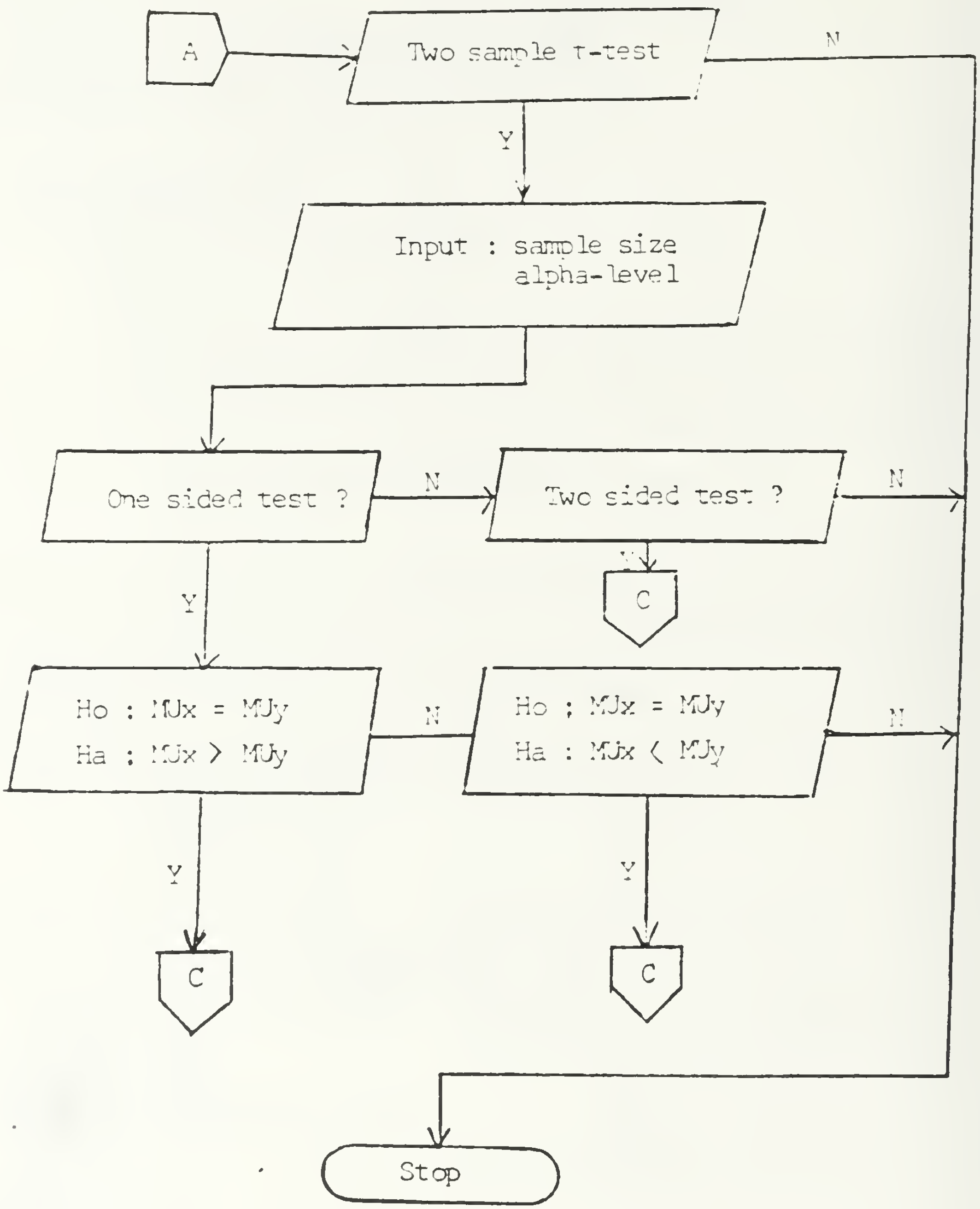


Figure 4.1b System flowchart for t-test power(cont'd.)

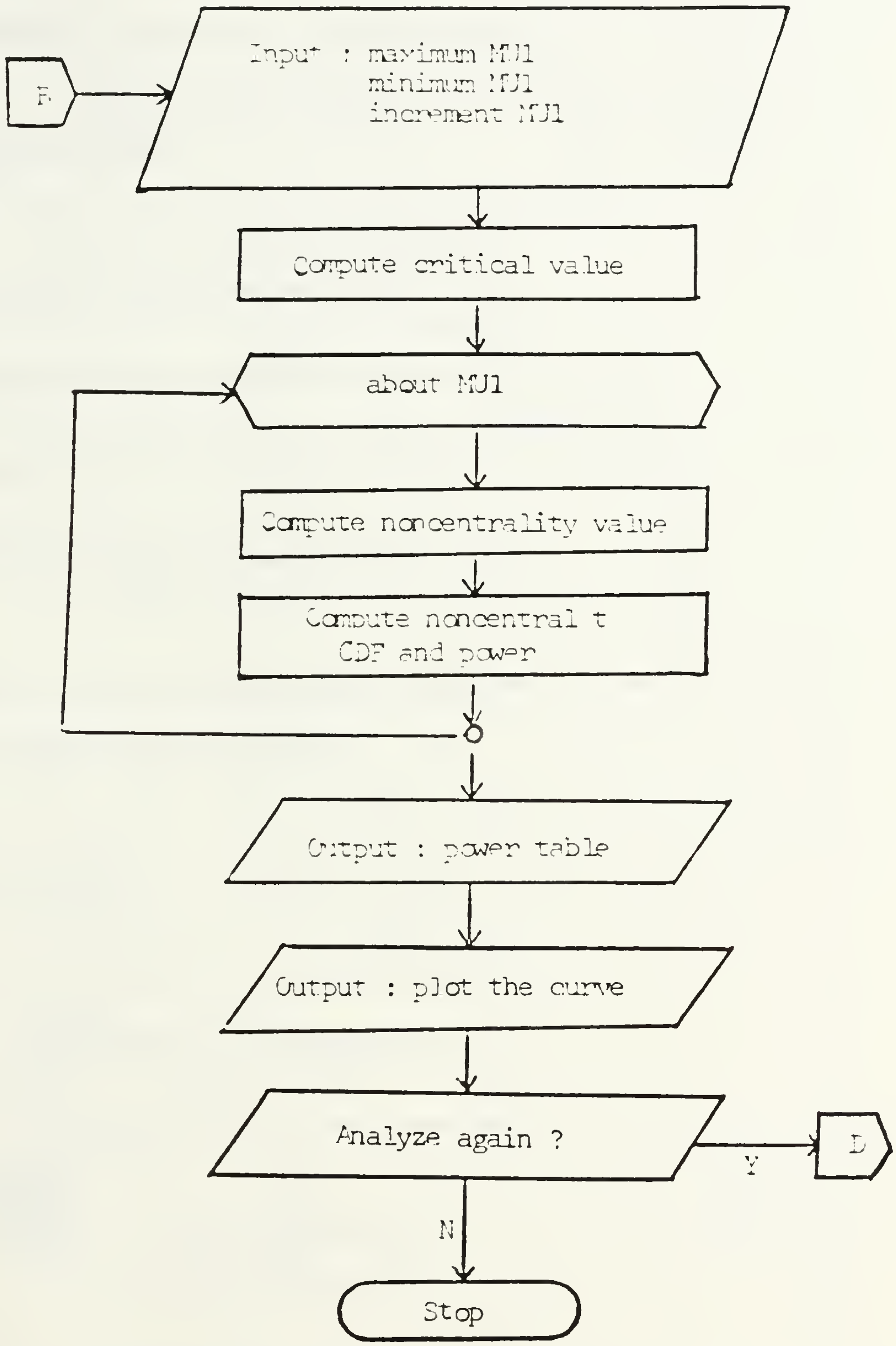


Figure 4.1c System flowchart for t-test power(cont'd.)

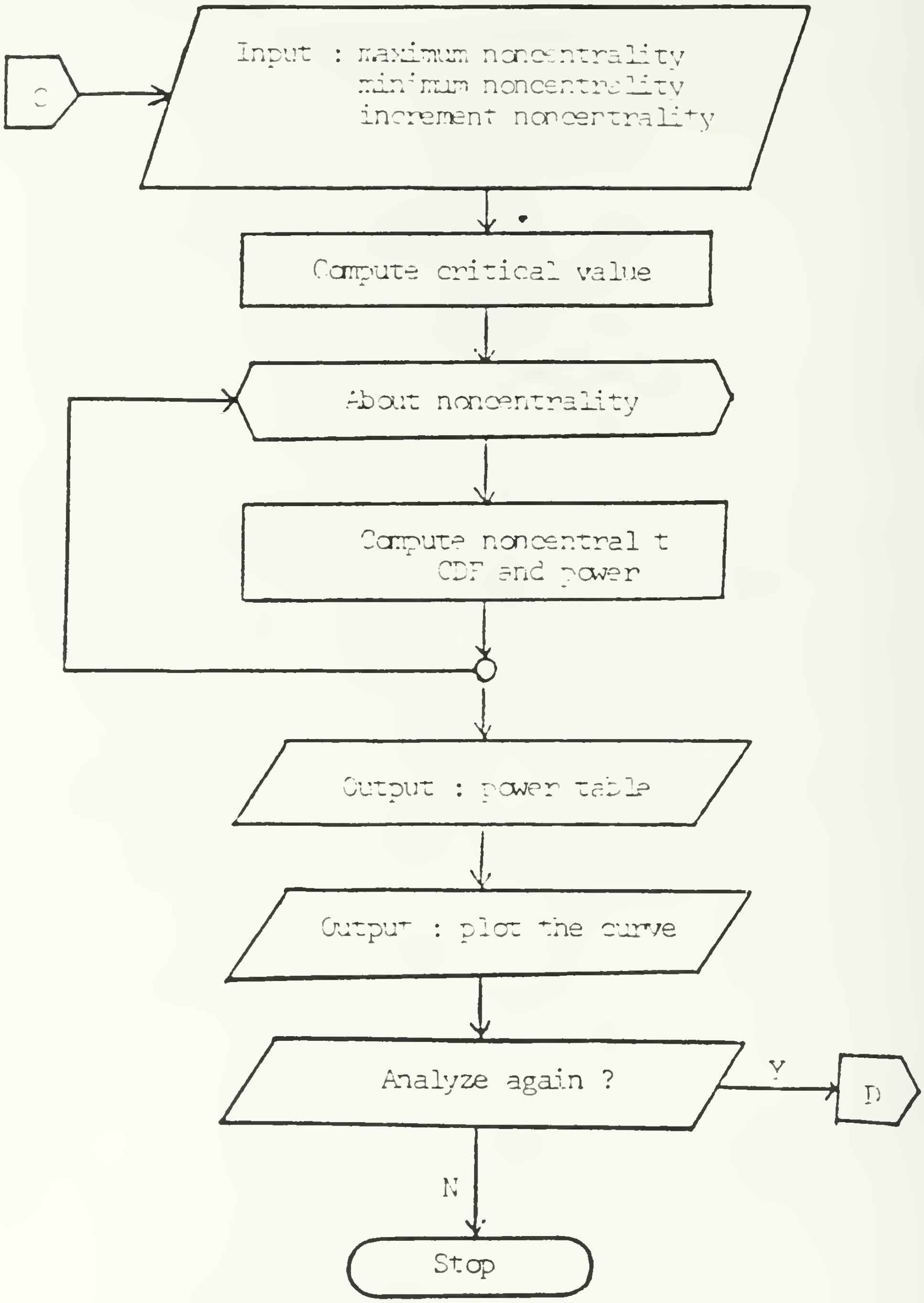


Figure 4.1d System flowchart for t-test power(cont'd.)

?

.05

Mu-zero value ?

?

40

Standard deviation value ?

?

2

Do you want to use one-sided test(y/n)?

y

Do you want to test Ho : mu $\leq$ mu0 vs Ha : mu > mu0 (y/n)?

n

Do you want to test Ho : mu $\geq$ mu0 vs Ha : mu < mu0 (y/n)?

y

Maximum mu1 value (the maximum value on X-axis)?

?

40

Minimum mu1 value (the minimum value on X-axis)?

(condition : mu1 must be less than mu0)

?

38

Increment mu1 value ?

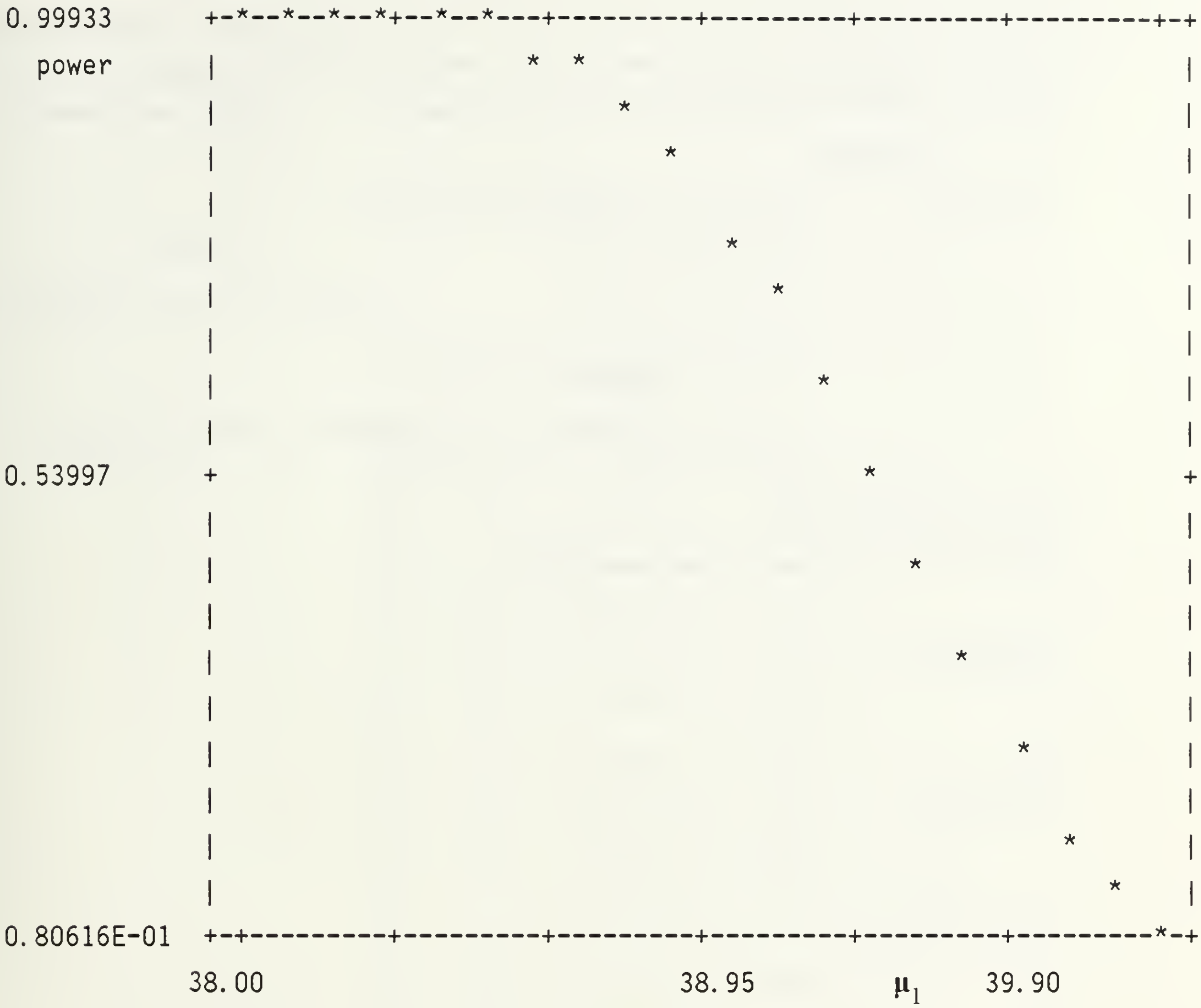
?

.1

d. Output

The screen output is as follows:

μ_1		power	
NN=	38.000	POWER=	0.99933
NN=	38.100	POWER=	0.99849
NN=	38.200	POWER=	0.99677
NN=	38.300	POWER=	0.99346
NN=	38.400	POWER=	0.98742
NN=	38.500	POWER=	0.97706
NN=	38.600	POWER=	0.96028
NN=	38.700	POWER=	0.93465
NN=	38.800	POWER=	0.89772
NN=	38.900	POWER=	0.84755
NN=	39.000	POWER=	0.78326
NN=	39.100	POWER=	0.70559
NN=	39.200	POWER=	0.61711
NN=	39.300	POWER=	0.52203
NN=	39.400	POWER=	0.42564
NN=	39.500	POWER=	0.33355
NN=	39.600	POWER=	0.25053
NN=	39.700	POWER=	0.17996
NN=	39.800	POWER=	0.12338
NN=	39.900	POWER=	0.08062



X-SCALE: "-" = 0.237E-01 UNITS

Y-SCALE: "|" = 0.153E-01 UNITS

2. Example of One sample two-sided t-test

a. Scenario

The supervisor in a rocket propellant plant is concerned with the burning rate of a rocket propellent. Standards specify that the mean burning rate must be 40 inches per second. For power computation, it is assumed that $\sigma^2 = 4.0$. The appropriate hypotheses are

$$Ho : \mu = 40$$
$$Ha : \mu \neq 40$$

The engineer decides to use a sample of n = 25 observations and α = .05. He would like to know the power of the t-test is for a range of possible population means. He decides to check means from 38 to 42 in .1 increments.

b. Inputs

INPUTS:

step 1: one sample two-sided t-test

step 2: $\mu_0 = 40$

step 3: $\sigma = 2.0$

step 4: $\alpha = .05$

step 5: sample size = 25

step 6: maximum $\mu_1 = 42$, minimum $\mu_1 = 38$, increment = .1

c. Start program

START PROGRAM:

Do you want to test one sample t-test(y/n)?

y

Sample size ?

?

25

Alpha-level ?

?

.05

Mu-zero value ?

?

40

Standard deviation value ?

?

2

Do you want to use one-sided test(y/n)?

n

Do you want to use two-sided test(y/n)?

y

Maximum mu1 value (the maximum value on X-axis)?

?

42

Minimum mu1 value (the minimum value on X-axis)?

(condition : mu1 must be less than mu-zero)

?

38

Increment mu1 value ?

?

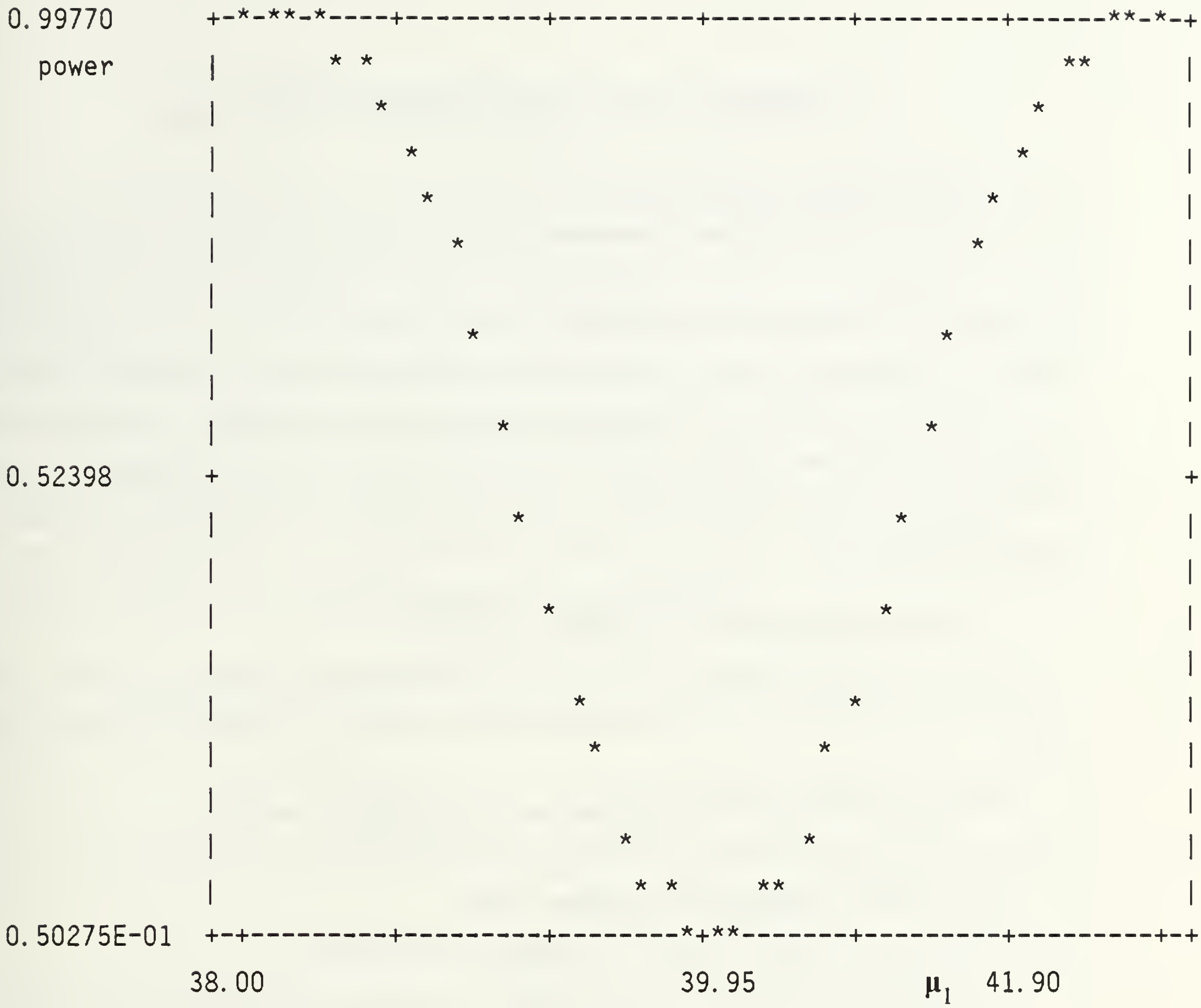
.1

d. Output

The screen output is as follows:

μ_1		POWER	
NN=	38.000	POWER=	0.99770
NN=	38.100	POWER=	0.99525
NN=	38.200	POWER=	0.99073
NN=	38.300	POWER=	0.98279
NN=	38.400	POWER=	0.96965
NN=	38.500	POWER=	0.94910
NN=	38.600	POWER=	0.91875
NN=	38.700	POWER=	0.87639
NN=	38.800	POWER=	0.82057
NN=	38.900	POWER=	0.75109
NN=	39.000	POWER=	0.66944
NN=	39.100	POWER=	0.57882
NN=	39.200	POWER=	0.48379
NN=	39.300	POWER=	0.38975
NN=	39.400	POWER=	0.30192
NN=	39.500	POWER=	0.22459
NN=	39.600	POWER=	0.16066
NN=	39.700	POWER=	0.11146
NN=	39.800	POWER=	0.07708
NN=	39.900	POWER=	0.05692
NN=	40.000	POWER=	0.05028
NN=	40.100	POWER=	0.05687
NN=	40.200	POWER=	0.07698
NN=	40.300	POWER=	0.11131
NN=	40.400	POWER=	0.16045

NN=	40.500	POWER=	0.22433
NN=	40.600	POWER=	0.30162
NN=	40.700	POWER=	0.38942
NN=	40.800	POWER=	0.48344
NN=	40.900	POWER=	0.57848
NN=	41.000	POWER=	0.66912
NN=	41.100	POWER=	0.75081
NN=	41.200	POWER=	0.82034
NN=	41.300	POWER=	0.87621
NN=	41.400	POWER=	0.91862
NN=	41.500	POWER=	0.94901
NN=	41.600	POWER=	0.96959
NN=	41.700	POWER=	0.98275
NN=	41.800	POWER=	0.99070
NN=	41.900	POWER=	0.99524



X-SCALE: "-"= 0.487E-01 UNITS

Y-SCALE: "|"= 0.158E-01 UNITS

V. DESCRIPTION OF THE POWER PROGRAM

A. PROCEDURE OVERVIEW

The interactive program included in this thesis is written in FORTRAN 77. It was written for use on an IBM370 from IBM3278 terminal. It is an interactive program. The command 'NONCENT' is all that is required to start this program. The executive file name initializes the virtual machine environment and asks the user questions about the choice of program and compilation requirements. It then activates the selected program. Both ANOVA and TTEST are interactive programs which will be discussed in detail in later chapters. The output from the programs is presented on the terminal screen.

B. INSTRUCTIONS FOR PROGRAM ACCESS

To start the program, make sure you have loaded (NONCEN, ANOVA, and TTEST) on your disk. In CMS (operating system mode), type 'NONCEN' and you will see:

Please provide the FILENAME for your VS FORTRAN program.

Now type the program name you want, for example 'TTEST', the response is:

Do you need to compile your program ? (y/n)

If you want to run, type 'Y' and your program will be loaded.

A detailed description of the ANOVA (Analysis of Variance Test) is given in Chapter III and of the t-test in Chapter IV.

Following the screen output from the selected test, the user will be asked some questions about the output:

Do you wish to BROWSE your output? (Y)

n

Print your output file? (Y)

n

Do you wish to XEDIT the program file? (Y/N)

n

Do you wish to run the program again? (Y)

n

Then return to CMS mode.

VI. SUMMARY AND CONCLUSIONS

Power considerations are useful in the design and assessment of statistical tests. The calculation of power usually involves noncentral distributions for which tables of probability are not available. A search was made for algorithms approximating the noncentral t, F and χ^2 distributions. Algorithms giving sufficient accuracy and making efficient use of computer resources have been implemented in this thesis. Listings of the FORTRAN code for these implementations are included.

An interactive program to compute and display power curves for several t-test and F-test situations has been developed. This program is user friendly and is described in this thesis. A listing of the program is provided. This program should be useful to researchers, experiment designers and statisticians.

APPENDIX A

AN INVERSE CENTRAL F APPROXIMATION

Let P be the central F complementary CDF, and let Q be the complementary standard normal CDF [Ref. 4: p. 947].

Define y_p by $Q(y_p)=p$, and F_p by $P(F_p \mid \nu_1, \nu_2)=p$.

Then $F_p \sim \exp(2w)$

where $w = w1-(w2)(w3)$

$$w1 = (y_p \surd (h+\lambda))/h$$

$$w2 = (1/(\nu_2 -1)-1/(\nu_1 -1))$$

$$w3 = (\lambda+(5/6)+2/(3h))$$

$$h = 2/((1/(\nu_1 -1)+1/(\nu_2 -1))$$

$$\lambda=(y_p{}^2 -3)/6$$

The program we used in the computation of critical values of F-tests is shown below.

```
**********************************************************************
*                                                                    *
*       APPROXIMATION  TO THE F- INVERSE DISTRIBUTION.              *
*       CALCULATE F-INVERSE GIVEN DOF1,DOF2,ALPHA.                  *
*       Q( F-ALPHA |DOF1,DOF2 )= ALPHA.                             *
*                                                                    *
**********************************************************************
      REAL ALPHA,NUMER,DENUM,LAM,XP,YP,FINVER,A,B
      DATA NF1,NF2,ALPHA/2,4,.01/
      DATA C0,C1,C2/2.515517,0.802853,0.010328/
      DATA D1,D2,D3/1.432788,0.189269,0.001308/
      B=NF1/2.
      A=NF2/2.
C
C     CALCULATE NORMAL QUANTILE.
C
      T=SQRT(ALOG(1./(ALPHA**2.)))
      NUMER=C0+C1*T+C2*T**2
      DENUM=1.+D1*T+D2*T**2.+D3*T**3.
      XP=T-(NUMER/DENUM)
C
C     CALCULATE THE INVERSE F-DISTRIBUTION.
C
      LAM=(XP**2-3.)/6.
      W2= (1./(2.*B-1.))-(1./(2.*A-1.))
      W4= (1./(2.*A-1.))+(1./(2.*B-1.))
      H=2.*W4**(-1.)
      W1=(LAM+(5./6.)-(2./(3.*H)))
      W3=(XP*(H+LAM)**.5)/H
      W=W3-W2*W1
      FINVER=EXP(2*W)
      WRITE(3,100) NF1,NF2,ALPHA,FINVER
 100  FORMAT('0','  NF1=',I3,'NF2=',I3,'ALPHA=',F5.4,'FINVERSE=',F6.3)
      STOP
      END
```

The following table shows some values obtained using this approximation, together with exact values of critical values of the F distribution. It can be seen in Table 8 that this approximation provides adequate accuracy for the computation of the critical value of the F-tests.

TABLE 8
APPROXIMATION AND EXACT CRITICAL VALUES OF F-TESTS

α	n1	n2	approx	exact
.05	4	4	6.581	6.39
	4	6	4.601	4.53
	4	10	3.525	3.48
	10	20	2.35	2.35
	20	30	1.932	1.93
	6	4	6.359	6.16
	10	6	4.112	4.06
	20	6	3.944	3.87
	30	20	2.041	2.04
	30	30	1.841	1.84
.10	4	4	4.150	4.11
	4	6	3.218	3.18
	4	10	2.648	2.61
	10	20	1.939	1.94
	20	30	1.668	1.67
	6	4	4.046	4.01
	10	6	2.952	2.94
	20	6	2.864	2.84
	30	20	1.739	1.74
	30	30	1.607	1.61

APPENDIX B
THE INVERSE CENTRAL T APPROXIMATION

We can use the asymptotic expansion for the Inverse t-CDF, as follows. Let Q be the complementary CDF of standard normal, and let P be the central t CDF [Ref. 4: p. 949].

If $P(t_p \mid \nu) = 1-2p$ and $Q(x_p) = p$

then

$$t_p \sim x_p + (g_1(x_p)/\nu) + (g_2(x_p)/\nu^2) + (g_3(x_p)/\nu^3) + (g_4(x_p)/\nu^4)$$

where $g_1(x) = .25(x^3 + x)$,

$g_2(x) = (1/96)(5x^5 + 16x^3 + 3x)$,

$g_3(x) = (1/384)(3x^7 + 19x^5 + 17x^3 - 15x)$, and

$g_4(x) = (1/92160)(79x^9 + 776x^7 + 1482x^5 - 1920x^3 - 945x)$.

The program we used in the computation of the critical value of the t-test is shown below.

```
*********************************************************************
*                                                                   *
*      APPROXIMATION  TO THE T- INVERSE DISTRIBUTION.               *
*      CALCULATE T-INVERSE GIVEN DOF ALPHA.                         *
*      Q( T-ALPHA | DOF )= ALPHA.                                   *
*                                                                   *
*********************************************************************
C
C     set initial conditions.
C
      REAL ALPHA,NUMER,DENUM,LAM,XP,G1,G2,G3,X,TINVER
      INTEGER N
      DATA N,ALPHA/9,.025/
      DATA C0,C1,C2/2.515517,0.802853,0.010328/
      DATA D1,D2,D3/1.432788,0.189269,0.001308/
C
C     CALCULATE NORMAL QUANTILE.
C
      ALPHA=.05
      T=SQRT(ALOG(1./(ALPHA**2.)))
      NUMER=C0+C1*T+C2*T**2
      DENUM=1.+D1*T+D2*T**2.+D3*T**3.
      XP=T-(NUMER/DENUM)
C
C     CALCULATE THE INVERSE T-DISTRIBUTION.
C
      X=XP
      G1=.25*(X**3+X)
      G2=(1./96.)*(5.*X**5+16.*X**3+3.*X)
      G3=(1./384.)*(3.*X**7+19.*X**3+17.*X**3-15.*X)
      G4=(1./92160.)*(79.*X**9+776.*X**7+1482.*X**5-1592.*X**3-945.*X)
      TP=X+(G1/N)+(G2/(N*N))+(G3/(N**3))+(G4/(N**4))
      TINVER=TP
      WRITE(3,100) N,ALPHA,TINVER
 100  FORMAT('0','  DOF=',I3,4X,'ALPHA=',F5.3,6X,'TINVERSE=',F9.5)
      STOP
      END
```

The following table shows some values obtained using this approximation together with exact values of critical values of the t distribution. It can be seen in Table 9 that this approximation

provides adequate accuracy for computing the critical regions of the t-tests.

TABLE 9
APPROXIMATION AND EXACT CRITICAL VALUES OF T-TESTS

DOF	α (Type I error)			
	.05		.01	
	exact	approx	exact	approx
2	2.919	2.867	6.964	6.475
4	2.132	2.126	3.746	3.701
6	1.943	1.942	3.142	3.130
8	1.833	1.833	2.821	2.818
10	1.812	1.823	2.764	2.761
12	1.783	1.783	2.681	2.680
14	1.761	1.761	2.624	2.624
16	1.746	1.746	2.583	2.583
18	1.734	1.734	2.552	2.552
20	1.725	1.725	2.528	2.528
25	1.708	1.708	2.485	2.485
35	1.690	1.690	2.438	2.438
45	1.690	1.690	2.412	2.413
65	1.669	1.669	2.385	2.385
85	1.663	1.663	2.371	2.371

APPENDIX C
AN APPROXIMATION OF THE NORMAL CDF

Let Z be the standard normal density function and let P be the corresponding CDF [Ref. 4: p. 939]. Then

$P(X) = \int Z(t)dt \sim 1-Z(x)(a_1t+a_2t^2+a_{3t}3)+ \varepsilon (x)$

where $t=1/(1+px)$

$| \varepsilon (x)| < .000001$

$p = .33267$, and $a_1 = .4361836$
$a_2 = - .1201676$
$a_3 = .9372980$

APPENDIX D
THE COMPUTATION OF NONCENTRAL F DISTRIBUTION

```
***********************  INFORMATION  **********************************
*                                                                      *
*       THE OBJECTIVE OF THIS PROGRAM IS TO CALCULATE NON-CENTRAL F    *
*       DISTRIBUTION (CDF,F-INVERSE).                                  *
*       THE CALCULATION METHOD IS NORMAL APPROXIMATION.                *
*                                                                      *
**********************  VARIABLE DEFINITION  ***************************
*                                                                      *
*       FVAL : X-VALUE DIMENSION                                       *
*       FCDF : NON-CENTRAL F CDF DIMENSION.                            *
*       FINVER : GIVEN X VALUE, FIND INVERSE VALUE.                    *
*       FLAM : NON-CENTRAL PARAMETER                                   *
*       FX : TO FIND INVERSE, GIVEN X VALUE.                           *
*       FSTAR : X VALUE VARIABLE.                                      *
*       NF1 : DEGREE OF FREEDOM 1.                                     *
*       NF2 : DEGREE OF FREEDOM 2.                                     *
*       CDF  : NORMAL DISTRIBUTION FUNCTION.                           *
*              P ( Z <= X)                                             *
*       CONST : 1 / SQRT ( 2 * PI ) IN NORMAL DISTRIBUTION FUNCTION.   *
*       B     : THE VALUE OF 'X' IN NORMAL DISTRIBUTION.               *
*                                                                      *
************************************************************************
C
C     SET INITIAL CONDITIONS.
C
      REAL FVAL(100),FCDF(100),FINVER,FLAM,FMAX,FX,FINIAL
      DATA NF1,NF2,FLAM,FMAX/2,4,75.0,90./
      DATA A1,A2,A3,P/.4361836,-.1201676,.9372980,.33267/
      DATA FSTAR/25.128/
         WRITE(3,24) NF1,NF2,FLAM
         WRITE(3,21)
         WRITE(3,22)
         WRITE(3,23)
         CONST  = 1.0 / SQRT (2.0 * 3.1415927)
C
C     CALCULATE THE NORMAL APPROXIMATION .
C
         C1=(NF1*FSTAR)/(NF1+FLAM)
         C2=1-(2./(9.*NF2))
         C3=(2.*(NF1+2.*FLAM))/(9.*(NF1+FLAM)**2.)
         CC1=(C1**(1./3.))*C2-(1-C3)
         CC2=((C3+(2./(9.*NF2))*(C1)**(2./3.))**.5)
         FX=CC1/CC2
         CDF = 0.0
         IF(FX .EQ.0) THEN
            CDF = 0.5
         ELSE IF(FX.GT.0) THEN
                T=1./(1.+P*FX)
                XSQU=(FX*FX)*.5
                ZPDF=CONST*EXP(-XSQU)
                CDF=1-ZPDF*(A1*T+A2*T**2+A3*T**3)
              ELSE
                FX=-FX
                T=1./(1.+P*FX)
                XSQU=(FX*FX)*.5
                ZPDF=CONST*EXP(-XSQU)
                CDF=ZPDF*(A1*T+A2*T**2+A3*T**3)
         END IF
              FVAL(FSTAR)=FSTAR
              FCDF(FSTAR)=CDF
         WRITE(3,250) FSTAR,,FCDF(FSTAR)
250      FORMAT('0',F7.3,8X,F8.5)
21       FORMAT('0',40('_'))
22       FORMAT('0',2X,'"X"-VALUE',8X,'F(Z <= X)')
23       FORMAT('0',40('_'))
24       FORMAT('0','DOF1=',I2,3X,'DOF2=',I2,3X,'NON-CENTRAL PARAMETER=
     &',F8.4)
         STOP
         END
```

APPENDIX E
THE COMPUTATION OF NONCENTRAL T DISTRIBUTION

```
***********************  INFORMATION  **********************************
*                                                                      *
*        THE OBJECTIVE OF THIS PROGRAM IS TO CALCULATE NON-CENTRAL T   *
*        DISTRIBUTIONS ( CDFs)                                         *
*        THE CALCULATION METHOD IS NORMAL APPROXIMATION.               *
*        (REF. SEVERO & ZELEN'S (1960) )                               *
*                                                                      *
**********************  VARIABLE DEFINITION  **************************
*                                                                      *
*        TVAL : X-VALUE DIMENSION                                      *
*        TCDF : NON-CENTRAL T CDF DIMENSION.                           *
*        TLAM : NON-CENTRALITY PARAMETER                               *
*        TX : TO FIND INVERSE, GIVEN X VALUE.                          *
*        TSTAR : X VALUE VARIABLE.                                     *
*        NF  : DEGREE OF FREEDOM                                       *
*        CDF  : NORMAL DISTRIBUTION FUNCTION.                          *
*               P ( Z <= X)                                            *
*        CONST : 1 / SQRT ( 2 * PI ) IN NORMAL DISTRIBUTION FUNCTION.  *
*        B      : THE VALUE OF 'X' IN NORMAL DISTRIBUTION.             *
*                                                                      *
************************************************************************
C
C     SET INITIAL CONDITIONS.
C
      REAL TVAL(1DD),TCDF(100),TINVER,TLAM,TMAX,TX,TINIAL
      DATA NF,TLAM/24,33.D78/
      DATA A1,A2,A3,P/.4361836,-.1201676,.9372980,.33267/
      DATA TSTAR/24.4949/
         WRITE(3,24) NF,TLAM
         WRITE(3,21)
         WRITE(3,22)
         WRITE(3,23)
         CONST  = 1.0 / SQRT (2.0 * 3.1415927)
C
C     CALCULATE THE NORMAL APPROXIMATION .
C
         C1=(1.-(1./(4.*NF)))*TSTAR-TLAM
         C2=SQRT(1.+(TSTAR**2/(2.*NF)))
         TX=C1/C2

         CDF = D.0
         IF(TX .EQ.0) THEN
         CDF = 0.5
         ELSE IF(TX.GT.D) THEN
              T=1./(1.+P*TX)
              XSQU=(TX*TX)*.5
              ZPDF=CONST*EXP(-XSQU)
              CDF=1-ZPDF*(A1*T+A2*T**2+A3*T**3)
              ELSE
              TX=-TX
              T=1./(1.+P*TX)
              XSQU=(TX*TX)*.5
              ZPDF=CONST*EXP(-XSQU)
              CDF=ZPDF*(A1*T+A2*T**2+A3*T**3)
         END IF
              TVAL(TSTAR)=TSTAR
              TCDF(TSTAR)=CDF
         WRITE(3,250) TSTAR,,TCDF(TSTAR)
 250      FORMAT('0',F7.3,8X,F8.5)
 21       FORMAT('0',40('_'))
 22       FORMAT('0',2X,'"X"-VALUE',8X,'F(Z <= X)')
 23       FORMAT('0',4D('_'))
 24       FORMAT('D','DOF=',I2,3X,'NON-CENTRAL PARAMETER=
     &',F8.4)
         STOP
         END
```

APPENDIX F
THE POWER OF ANOVA PROGRAM LIST

```
*******************************************************************
*                                                                 *
*    POWER ALGORITHM ( ANALYSIS OF VARIANCE )                     *
*                                                                 *
*                                                                 *
*  DIRECTED BY : PROFESSOR  DONALD.R.BARR                         *
*                                                                 *
*  WRITTEN BY : HUR, SEONG PIL            JULY 1986               *
*             DEPARTMENT OF  O.R.        NAVAL POSTGRADUATE SCHOOL *
*                                                                 *
*                                                                 *
*    PROGRAM USES THE POWER PLOT TO TEST FOR ANALYSIS OF VARIANCE. *
*                                                                 *
*             (A) ONE-WAY ANOVA                                   *
*                 (1)  SAMPLE SIZE VS POWER                       *
*                 (2)  NUMBER OF TREATMENT VS POWER               *
*                 (3)  ALPHA-LEVEL VS POWER                       *
*                 (4)  NONCENTRALITY PARAMETER VS POWER           *
*                                                                 *
*             (B) MULTI-WAY ANOVA                                 *
*                 (1)  NUMBER OF REPLICATES VS POWER              *
*                 (3)  ALPHA-LEVEL VS POWER                       *
*                 (4)  NONCENTRALITY PARAMETER VS POWER           *
*  NOTE : subroutine PLOTT is NONIMSL subroutine library.         *
*                                                                 *
*******************************************************************
      REAL NN(200),POWER(200),NON,FINV,NONCEN,FPOW,FX,ALPHA,CDF,CONST
      REAL MAXDEL,MINDEL,INCDEL,STDS,PICON,MINSD,MAXSD,INCSD,MAXALP
      REAL MINALP,INCRAL
      INTEGER MF(20),MAXN,MINN,INCRN,MAXK,MINK,INCRK,JJ,K,N
      CHARACTER*1 ANS
C_________________________________________________________________
C
C     START ONE-WAY OR MULTI-WAY ANALYSIS OF VARIANCE.
C
C_________________________________________________________________
500   PRINT*,'DO YOU WANT TO USE ONE-WAY ANOVA (Y/N)?'
      READ(5,7) ANS
 7    FORMAT(A1)
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 1000
      PRINT*,'DO YOU WANT TO PLOT N( # OF OBSERVATION PER TREATMENT)
     & VS POWER ( Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 10
      PRINT*,'DO YOU WANT TO PLOT K ( NUMBER OF  TREATMENT ) VS POWER
     & ( Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 20
      PRINT*,'DO YOU WANT TO PLOT ALPHA-LEVEL VS POWER (Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 30
      PRINT*,'DO YOU WANT TO PLOT NONCENTRALITY VALUE VS POWER (Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 40
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 500
      GO TO 999
C_________________________________________________________________
C
C     ANALYZE NUMBER OF SAMPLE SIZE PER TREATMENT VS POWER IN ONE-WAY
C             ANALYSIS OF VARIANCE.
C
C_________________________________________________________________
10    PRINT*,'NUMBER  OF K( # OF TREATMENT )?'
      READ (5,*) K
      PRINT*,'ALPHA- LEVEL?'
      READ (5,*) ALPHA
      PRINT*,' STANDARDIZED SQUARED DISTANCE VALUE ?'
      READ (5,*) STDS
```

```
      PRINT*,' MAXIMUM N VALUE ( THE MAXIMUM VALUE ON X-AXIS )?'
      READ (5,*) MAXN
      PRINT*,'MINIMUM N VALUE ( THE MINIMUM VALUE ON X-AXIS )?
     & (CONDITION :N MUST BE MORE THAN 2 )'
      READ (5,*) MINN
      PRINT*,'INCREMENT  N VALUE ?'
      READ (5,*) INCRN
      WRITE(6,95)
 95   FORMAT(1X,'DOF1  DOF2  ALPHA-LEVEL  F-INVERSE  NONCENTRAL  POWER')
      NF1=K-1
      JJ=1
      DO 110 N=MINN,MAXN,INCRN
      NF2=K*(N-1)
C_______________________________________________________________________
C
C     COMPUTE CENTRAL F-INVERSE GIVEN DOF1,DOF2 AND ALPHA-LEVEL.
C
C
C_______________________________________________________________________
      FINV=FINVER (NF1,NF2,ALPHA)
C_______________________________________________________________________
C
C     COMPUTE NONCENTRALITY PARAMETER AS SAMPLE SIZE CHANGE.
C
C_______________________________________________________________________
      NONCEN=N*STDS
C_______________________________________________________________________
C
C     COMPUTE POWER USING NONCENTRAL F-DISTRIBUTION (CDF) GIVEN DOF1,
C        DOF2,CENTRAL F-INVERSE AND NONCENTRALITY VALUE.
C
C_______________________________________________________________________
      POW=FPO(ALPHA,NF1,NF2,FINV,NONCEN)
      WRITE(6,15) NF1,NF2,ALPHA,FINV,NONCEN,POW
 15   FORMAT('0',I5,3X,I5,2X,2F10.5,1X,F12.2,1X,F10.5)
      NN(JJ)=N
      POWER(JJ)=POW
      JJ=JJ+1
 110  CONTINUE
      JJ=JJ-1
      PRINT*,'NN        POWER'
      DO 76 N=1,JJ
      WRITE(6,75)  NN(N),POWER(N)
 75   FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
 76   CONTINUE
C_______________________________________________________________________
C
C     PLOT NUMBER OF SAMPLE SIZE PER TREATMENT VS POWER IN ONE-WAY
C      ANALYSIS OF VARIANCE USING LIBRARY SUBROUTINE.
C
C_______________________________________________________________________
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 500
      GO TO 999
C_______________________________________________________________________
C
C   ANALYZE NUMBER OF TREATMENT VS POWER IN ONE-WAY ANALYSIS OF VARIANCE
C
C_______________________________________________________________________
20    PRINT*,'# OF OBSERVATIONS PER TREATMENT (N=  ) ?'
      READ (5,*) N
      PRINT*,'ALPHA LEVEL (REAL VALUE) ?'
      READ (5,*) ALPHA
      PRINT*,'WHAT STANDARDIZED SQUARED DISTANCE VALUE ?'
      READ (5,*) STDS
      PRINT*,'MAXIMUM  K (# OF TREATMENTS) RANGE ?'
      READ (5,*) MAXK
      PRINT*,'MINIMUM K (# OF TREATMENTS) RANGE (K MUST BE MORE THAN 2)?'
      READ (5,*) MINK
      PRINT*,'INCREMENT  K VALUE ?'
      READ (5,*) INCRK
      WRITE(6,195)
195   FORMAT(1X,'DOF1  DOF2  ALPHA-LEVEL  F-INVERSE  NONCENTRAL  POWER')
      JJ=1
      DO 111 K=MINK,MAXK,INCRK
      NF1=K-1
      NF2=K*(N-1)
      FINV=FINVER (NF1,NF2,ALPHA)
      NONCEN=N*STDS
      POW=FPO(ALPHA,NF1,NF2,FINV,NONCEN)
      WRITE(6,115) NF1,NF2,ALPHA,FINV,NONCEN,POW
 115  FORMAT('0',I5,3X,I5,2X,2F10.5,1X,F12.2,1X,F10.5)
      NN(JJ)=K
      POWER(JJ)=POW
```

```
      JJ=JJ+1
 111  CONTINUE
      PRINT*,'NN     POWER'
      DO 176 N=1,JJ
      WRITE(6,175)  NN(N),POWER(N)
175   FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
176   CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 500
      GO TO 999
C______________________________________________________________________
C
C   ANALYZE ALPHA-LEVEL VS POWER IN ONE-WAY ANALYSIS OF VARIANCE.
C
C______________________________________________________________________
30    PRINT*,'# OF OBSERVATIONS PER TREATMENT (N= ) ?'
      READ (5,*) N
      PRINT*,'# OF TREATMENTS (K= )?'
      READ (5,*) K
      PRINT*,' STANDARDIZED SQUARED DISTANCE VALUE ?'
      READ (5,*) STDS
      PRINT*,'MAXIMUM  ALPHA  RANGE ?'
      READ (5,*) MAXALP
      PRINT*,'MINIMUM ALPHA  RANGE  ?'
      READ (5,*) MINALP
      PRINT*,'INCREMENT  ALPHA VALUE ?'
      READ (5,*) INCRAL
      WRITE(6,295)
295   FORMAT(1X,'DOF1  DOF2  ALPHA-LEVEL  F-INVERSE  NONCENTRAL  POWER')
      JJ=1
      NF1=K-1
      NF2=K*(N-1)
      DO 211 ALPHA=MINALP,MAXALP,INCRAL
      FINV=FINVER (NF1,NF2,ALPHA)
      NONCEN=N*STDS
      POW=FPO(ALPHA,NF1,NF2,FINV,NONCEN)
      WRITE(6,215) NF1,NF2,ALPHA,FINV,NONCEN,POW
 215  FORMAT('0',I5,3X,I5,2X,2F10.5,1X,F12.2,1X,F10.5)
      NN(JJ)=ALPHA
      POWER(JJ)=POW
      JJ=JJ+1
 211  CONTINUE
      JJ=JJ-1
       PRINT*,'NN       POWER'
       DO 276 I =1,JJ
       WRITE(6,275)  NN(JJ),POWER(JJ)
 275   FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
 276   CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 500
      GO TO 999
C______________________________________________________________________
C
C        ANALYZE NONCENTRALITY PARAMETER VS POWER IN ONE-WAY ANALYSIS OF
C                 VARIANCE
C
C______________________________________________________________________
40    PRINT*,'# OF OBSERVATIONS PER TREATMENT (N= ) ?'
      READ (5,*) N
      PRINT*,'# OF TREATMENTS( K=)?'
      READ (5,*) K
      PRINT*,'ALPHA-LEVEL  ?'
      READ (5,*) ALPHA
      PRINT*,'MAXIMUM  STANDARDIZED SQUARED DISTANCE VALUE RANGE ?'
      READ (5,*) MAXSD
      PRINT*,'MINIMUM  STANDARDIZED SQUARED DISTANCE VALUE RANGE ?'
      READ (5,*) MINSD
      PRINT*,'INCREMENT STANDARDIZED SQUARED DISTANCE VALUE RANGE ?'
      READ (5,*) INCSD
      WRITE(6,495)
495   FORMAT(1X,'DOF1  DOF2  ALPHA-LEVEL  F-INVERSE  NONCENTRAL  POWER')
      JJ=1
      NF1=K-1
      NF2=K*(N-1)
      DO 411 STDS=MINSD,MAXSD,INCSD
      FINV=FINVER (NF1,NF2,ALPHA)
      NONCEN=N*STDS
      POW=FPO(ALPHA,NF1,NF2,FINV,NONCEN)
      WRITE(6,415) NF1,NF2,ALPHA,FINV,NONCEN,POW
 415  FORMAT('0',I5,3X,I5,2X,2F10.5,1X,F12.2,1X,F10.5)
      NN(JJ)=NONCEN
```

```
      POWER(JJ)=POW
      JJ=JJ+1
 411  CONTINUE
      JJ=JJ-1
      DO 476 I =1,JJ
      WRITE(6,475)  NN(I),POWER(I)
 475  FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
 476  CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 500
      GO TO 999
C_____________________________________________________________________
C
C        ANALYZE MULTI-WAY ANALYSIS OF VARIANCE.
C
C_____________________________________________________________________
1000  PRINT*,'HOW MANY WAYS ANOVA CLASSIFICATION?'
      READ(5,*) M
      DO 600 I=1,M
      PRINT*,'FOR',I,'-FACTOR LEVEL ,HOW MANY LEVELS ?'
      READ(5,*) MF(I)
600   CONTINUE
C_____________________________________________________________________
C
C        ANALYZE MODEL WITH FACTOR-LEVEL IN MULTI-WAY ANOVA.
C
C_____________________________________________________________________
      PRINT*,'IS THERE ANY INTERACTION ?(Y/N)'
      READ(5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 3100
      ISUM=0
      ITOT=1
      DO 800 I=1,M
         ISUM=ISUM+(MF(I)-1)
         ITOT=ITOT*MF(I)
800   CONTINUE
      IERDF=ITOT-ISUM-1
C_____________________________________________________________________
C
C        COMPUTE MINIMUM NUMBER OF REPLICATE PER TREATMENT.
C
C_____________________________________________________________________
      MINR=MINIR(ITOT,ISUM)
      PRINT*,'AT LEAST MINIMUM NUMBER OF REPLICATE PER TREATMENT =',MINR
      PRINT*,'WHICH FACTOR ANALYSIS (IF YOU WANT 3 RD FACTOR,PRESS 3)?'
      READ(5,*) I
      NF1=MF(I)-1
      PICON=FLOAT(ITOT)/FLOAT(MF(I))
      GO TO 2000
C_____________________________________________________________________
C
C        ANALYZE MODEL WITH FACTOR-LEVEL AND 2-WAY INTERACTION IN
C                MULTI-WAY ANOVA.
C
C_____________________________________________________________________
3100  PRINT*,'ARE THERE ONLY 2-WAY INTERACTION AND FACTOR-LEVEL?(Y/N)'
      READ(5,7) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 2150
      ISUM1=0
      ISUM2=0
      ITOT=1
      DO 850 I=1,M
         ISUM1=ISUM1+(MF(I)-1)
         ITOT=ITOT*MF(I)
850   CONTINUE
      DO 860 I=1,M-1
         DO 870 J=I+1,M
         ISUM2=ISUM2+(MF(I)-1)*(MF(J)-1)
870   CONTINUE
860   CONTINUE
      ISUM=ISUM1+ISUM2
      IERDF=ITOT-ISUM-1
      MINR=MINIR(ITOT,ISUM)
      PRINT*,'AT LEAST MINIMUM NUMBER OF REPLICATE PER TREATMENT =',MINR
      PRINT*,'DO YOU WANT TO USE ONLY FACTOR-LEVEL (Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 880
      PRINT*,'WHICH FACTOR ANALYSIS (IF YOU WANT 3 RD FACTOR,THEN
     &PRESS 3)?'
      READ(5,*) I
      NF1=MF(I)-1
      PICON=FLOAT(ITOT)/FLOAT(MF(I))
      GO TO 2000
```

```
880   PRINT*,'WHICH INTERACTION ANALYSIS?( IF YOU WANT 2 AND 3 WAY INTER
     &ACTION,THEN PRESS 2  3 ).'
      READ(5,*) I,J
      NF1=(MF(I)-1)*(MF(J)-1)
      PICON=FLOAT(ITOT)/FLOAT(MF(I)*MF(J))
      GO TO 2000
C_______________________________________________________________________
C
C        ANALYZE MODEL WITH FACTOR-LEVEL AND 3-WAY INTERACTION IN
C                MULTI-WAY ANOVA.
C
C_______________________________________________________________________
2150  PRINT*,'ARE THERE ONLY 3-WAY INTERACTION AND FACTOR-LEVEL ?(Y/N)'
      READ(5,7) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 2200
      ISUM1=0
      ISUM3=0
      ITOT=1
      DO 871 I=1,M
         ISUM1=ISUM1+(MF(I)-1)
         ITOT=ITOT*MF(I)
871   CONTINUE
      DO 872 I=1,M-2
         DO 873 J=I+1,M-1
            DO 874 K=J+1,M
         ISUM3=ISUM3+(MF(I)-1)*(MF(J)-1)*(MF(K)-1)
874   CONTINUE
873   CONTINUE
872   CONTINUE
      ISUM=ISUM1+ISUM3
      IERDF=ITOT-ISUM-1
      MINR=MINIR(ITOT,ISUM)
      PRINT*,'AT LEAST MINIMUM NUMBER OF REPLICATE PER TREATMENT =',MINR
      PRINT*,'DO YOU WANT TO ANALYSIS ONLY FACTOR-LEVEL ?(Y/N)'
      READ(5,7) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 875
      PRINT*,'WHICH FACTOR ANALYSIS (IF YOU WANT 3 RD FACTOR,PRESS 3)?'
      READ(5,*) I
      NF1=MF(I)-1
      PICON=FLOAT(ITOT)/FLOAT(MF(I))
      GO TO 2000
875   PRINT*,'DO YOU WANT TO ANALYSIS 3-WAY INTERACTION ?(Y/N)'
      READ(5,7) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 500
      PRINT*,'WHICH INTERACTION ANALYSIS?( IF YOU WANT 2 AND 3 AND 4 INT
     &ERACTION ,THEN PRESS 2  3  4 ).'
      READ(5,*) I,J,K
      NF1=(MF(I)-1)*(MF(J)-1)*(MF(K)-1)
      PICON=FLOAT(ITOT)/FLOAT(MF(I)*MF(J)*MF(K))
      GO TO 2000
C_______________________________________________________________________
C
C        ANALYZE MODEL WITH FACTOR-LEVEL ,2-WAY INTERACTION IN AND
C                3-WAY INTERACTION IN MULTI-WAY ANOVA.
C
C_______________________________________________________________________
2200  PRINT*,'ARE THERE  2-WAY INTERACTIONS AND 3-WAY INTERACTIONS AND
     &FACTER-LEVEL (Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 999
      ISUM1=0
      ISUM2=0
      ISUM3=0
      ITOT=1
      DO 950 I=1,M
         ISUM1=ISUM1+(MF(I)-1)
         ITOT=ITOT*MF(I)
950   CONTINUE
      DO 960 I=1,M-1
         DO 970 J=I+1,M
         ISUM2=ISUM2+(MF(I)-1)*(MF(J)-1)
970   CONTINUE
960   CONTINUE
      DO 965 I=1,M-2
         DO 975 J=I+1,M-1
            DO 985 K=J+1,M
         ISUM3=ISUM3+(MF(I)-1)*(MF(J)-1)*(MF(K)-1)
985   CONTINUE
975   CONTINUE
965   CONTINUE
      ISUM=ISUM1+ISUM2+ISUM3
      IERDF=ITOT-ISUM-1
      MINR=MINIR(ITOT,ISUM)
      PRINT*,'AT LEAST MINIMUM NUMBER OF REPLICATE PER TREATMENT =',MINR
      PRINT*,'DO YOU WANT TO ANALYSIS ONLY FACTOR-LEVEL (Y/N)?'
```

```
      READ(5,7) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 990
      PRINT*,'WHICH FACTOR ANALYSIS (IF YOU WANT 3 RD FACTOR,PRESS 3)?'
      READ(5,*) I
      NF1=MF(I)-1
      PICON=FLOAT(ITOT)/FLOAT(MF(I))
      GO TO 2000
990   PRINT*,'DO YOU WANT TO ANALYZE 2-WAY INTERACTION (Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 995
      PRINT*,'WHICH INTERACTION ANALYSIS ?( IF YOU WANT 2 AND 3 WAY INTE
     &ACTION,THEN PRESS 2  3 ).'
      READ(5,*) I,J
      NF1=(MF(I)-1)*(MF(J)-1)
      PICON=FLOAT(ITOT)/FLOAT(MF(I)*MF(J))
      GO TO 2000
995   PRINT*,'DO YOU WANT TO ANALYZE 3-WAY INTERACTION (Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 500
      PRINT*,'WHICH INTERACTION ANALYSIS?( IF YOU WANT 2 AND 3 AND 4 INT
     &ERACTION ,THEN PRESS 2  3  4 ).'
      READ(5,*) I, J, K
      NF1=(MF(I)-1)*(MF(J)-1)*(MF(K)-1)
      PICON=FLOAT(ITOT)/FLOAT(MF(I)*MF(J)*MF(K))
      GO TO 2000
2000  PRINT*,'DO YOU WANT TO PLOT R( # OF REPLICATE PER CELL)
     &VS POWER ( Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 4100
      PRINT*,'DO YOU WANT TO PLOT NONCENTRALITY VS POWER (Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 4200
      PRINT*,'DO YOU WANT TO PLOT ALPHA-LEVEL VS POWER (Y/N)?'
      READ(5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 2300
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 500
      GO TO 999
C______________________________________________________________________
C
C     ANALYZE NUMBER OF SAMPLE SIZE PER TREATMENT VS POWER IN MULTI-WAY
C             ANALYSIS OF VARIANCE.
C
C______________________________________________________________________
4100  PRINT*,'ALPHA LEVEL?'
      READ (5,*) ALPHA
      PRINT*,'STANDARDIZED SQUARED DISTANCE VALUE ?'
      READ (5,*) STDS
      PRINT*,'MAXIMUM R VALUE ( THE MAXIMUM VALUE ON X-AXIS )?'
      READ (5,*) MAXR
      PRINT*,' MINIMUM R VALUE ( MUST BE MORE THAN',MINR,' )?'
      READ (5,*) MINR
      PRINT*,'INCREMENT R VALUE ?'
      READ (5,*) INCRR
      WRITE(6,4095)
4095  FORMAT(1X,'DOF1  DOF2  ALPHA-LEVEL  F-INVERSE  NONCENTRAL  POWER')
      JJ=1
      DO 4110 N=MINR,MAXR,INCRR
      NF2=N*ITOT-ISUM-1
      FINV=FINVER (NF1,NF2,ALPHA)
      NONCEN=N*PICON*STDS
      POW=FPO(ALPHA,NF1,NF2,FINV,NONCEN)
      WRITE(6,4015) NF1,NF2,ALPHA,FINV,NONCEN,POW
4015  FORMAT('0',I5,3X,I5,2X,2F10.5,1X,F12.2,1X,F10.5)
      NN(JJ)=N
      POWER(JJ)=POW
      JJ=JJ+1
4110  CONTINUE
      JJ=JJ-1
      PRINT*,'NN       POWER'
      DO 4076 N=1,JJ
      WRITE(6,4075)  NN(N),POWER(N)
4075  FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
4076  CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 500
      GO TO 999
C______________________________________________________________________
C
C     ANALYZE NONCENTRALITY PARAMETER VS POWER IN MULTI-WAY
C             ANALYSIS OF VARIANCE.
C
```

```
C_____________________________________________________________________
4200  PRINT*,'HOW MANY NUMNER OF REPLICATE (MUST BE MORE THAN',MINR,
     &')?'
      READ(5,*) IRR
      NF2=IRR*ITOT-ISUM-1
      PRINT*,'ALPHA-LEVEL  ?'
      READ (5,*) ALPHA
      PRINT*,'MAXIMUM  STANDARDIZED SQUARED DISTANCE VALUE RANGE ?'
      READ (5,*) MAXSD
      PRINT*,'MINIMUM  STANDARDIZED SQUARED DISTANCE  VALUE RANGE ?'
      READ (5,*) MINSD
      PRINT*,'INCREMENT STANDARDIZED SQUARED DISTANCE VALUE RANGE ?'
      READ (5,*) INCSD
      WRITE(6,4495)
4495  FORMAT(1X,'DOF1  DOF2  ALPHA-LEVEL  F-INVERSE  NONCENTRAL  POWER')
      JJ=1
      DO 4911 STDS=MINSD,MAXSD,INCSD
      FINV=FINVER (NF1,NF2,ALPHA)
      NONCEN=IRR*PICON*STDS
      POW=FPO(ALPHA,NF1,NF2,FINV,NONCEN)
      WRITE(6,4915) NF1,NF2,ALPHA,FINV,NONCEN,POW
4915  FORMAT('0',I5,3X,I5,2X,2F10.5,1X,F12.2,1X,F10.5)
      NN(JJ)=NONCEN
      POWER(JJ)=POW
      JJ=JJ+1
4911  CONTINUE
      JJ=JJ-1
      DO 1476 I =1,JJ
      WRITE(6,1475)  NN(I),POWER(I)
1475  FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
1476  CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 500
      GO TO 999
C_____________________________________________________________________
C
C     ANALYZE ALPHA-LEVEL PARAMETER VS POWER IN MULTI-WAY
C             ANALYSIS OF VARIANCE.
C
C_____________________________________________________________________
2300  PRINT*,'HOW MANY NUMNER OF REPLICATE (MUST BE MORE THAN',MINR,
     &')?'
      READ(5,*) IRR
      NF2=IRR*ITOT-ISUM-1
      READ (5,*) K
      PRINT*,'STANDARDIZED SQUARED DISTANCE VALUE ?'
      READ (5,*) STDS
      PRINT*,'MAXIMUM  ALPHA  RANGE ?'
      READ (5,*) MAXALP
      PRINT*,'MINIMUM ALPHA  RANGE  ?'
      READ (5,*) MINALP
      PRINT*,'INCREMENT  ALPHA VALUE ?'
      READ (5,*) INCRAL
      WRITE(6,1295)
1295  FORMAT(1X,'DOF1  DOF2  ALPHA-LEVEL  F-INVERSE  NONCENTRAL  POWER')
      JJ=1
      NF1=K-1
      NF2=K*(N-1)
      DO 1211 ALPHA=MINALP,MAXALP,INCRAL
      FINV=FINVER (NF1,NF2,ALPHA)
      NONCEN=IRR*PICON*STDS
      POW=FPO(ALPHA,NF1,NF2,FINV,NONCEN)
      WRITE(6,1215) NF1,NF2,ALPHA,FINV,NONCEN,POW
1215  FORMAT('0',I5,3X,I5,2X,2F10.5,1X,F12.2,1X,F10.5)
      NN(JJ)=ALPHA
      POWER(JJ)=POW
      JJ=JJ+1
1211  CONTINUE
      JJ=JJ-1
      PRINT*,'NN(JJ)',MINK,MAXK,INCRK
      DO 1276 I =1,JJ
      WRITE(6,1275)  NN(JJ),POWER(JJ)
1275  FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
1276  CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,7) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 500
      GO TO 999
999   STOP
      END
**********************************************************************
*                                                                    *
```

```
*       APPROXIMATION  TO THE F- INVERSE DISTRIBUTION.                  *
*       CALCULATE F-INVERSE GIVEN DOF1,DOF2,ALPHA.                      *
*       Q( F-ALPHA |DOF1,DOF2 )= ALPHA.                                 *
*                                                                       *
*************************************************************************
      FUNCTION FINVER(N1,N2,ALP)
      REAL ALP,NUMER,DENUM,LAM,XP,YP,A,B,FIN1
      DATA C0,C1,C2/2.515517,0.802853,0.010328/
      DATA D1,D2,D3/1.432788,0.189269,0.001308/
      B=N1/2.
      A=N2/2.
C________________________________________________________________________
C
C     CALCULATE NORMAL QUANTILE.
C
C________________________________________________________________________
      T=SQRT(ALOG(1./(ALP**2.)))
      NUMER=C0+C1*T+C2*T**2
      DENUM=1.+D1*T+D2*T**2+D3*T**3
      XP=T-(NUMER/DENUM)
C________________________________________________________________________
C
C     CALCULATE THE INVERSE F-DISTRIBUTION.
C
C________________________________________________________________________
      LAM=(XP**2-3.)/6.
      W2= (1./(2.*B-1.))-(1./(2.*A-1.))
      W4= (1./(2.*A-1.))+(1./(2.*B-1.))
      H=2.*W4**(-1.)
      W1=(LAM+(5./6.)-(2./(3.*H)))
      W3=(XP*(H+LAM)**.5)/H
      W=(W3-W2*W1)*2
              IF(W.LT.174) THEN
                   FIN1=EXP(W)
              ELSE
                   FIN1=1000
              ENDIF
      FINVER=FIN1
      RETURN
      END
*************************************************************************
*                                                                       *
*    GIVEN DOF1,DOF2,FINV,NON-CENTRAL PARAMETER,                        *
*         THEN COMPUTE POWER.                                           *
*            ( I.E.  CALCULATE NON-CENTRAL F-DISTRIBUTION ( C.D.F.)     *
*                                                                       *
*************************************************************************
      FUNCTION   FPO(ALP,N1,N2,FIN,NON)
      REAL NON
      INTEGER N1,N2
      DATA A1,A2,A3,P/.4361836,-.1201676,.9372980,.33267/
         CONST  = 1.0 / SQRT (2.0 * 3.1415927)
C________________________________________________________________________
C
C     CALCULATE THE NORMAL APPROXIMATION .
C
C________________________________________________________________________
         C1=(N1*FIN)/(N1+NON)
         C2=1-(2./(9.*N2))
         C3=(2.*(N1+2.*NON))/(9.*(N1+NON)**2)
         CC1=(C1**(1./3.))*C2-(1-C3)
         CC2=((C3+(2./(9.*N2))*(C1)**(2./3.))**.5)
         FX=CC1/CC2
C________________________________________________________________________
C
C     CALCULATE THE NONCENTRAL F-DISTRIBUTION USING NORMAL CDF.
C
C________________________________________________________________________

         CDF = 0.0
         IF(FX .EQ.0) THEN
         CDF = 0.5
         ELSE IF(FX.GT.0) THEN
              T=1./(1.+P*FX)
              XSQU=(FX*FX)*.5
              IF(XSQU.LT.174) THEN
                   ZPDF=CONST*EXP(-XSQU)
              ELSE
                   ZPDF=0
              ENDIF
C             ZPDF=CONST*EXP(-XSQU)
              CDF=1-ZPDF*(A1*T+A2*T**2+A3*T**3)
              ELSE
              FX=-FX
              T=1./(1.+P*FX)
```

```
          XSQU=(FX*FX)*.5
          IF(XSQU.LT.174) THEN
               ZPDF=CONST*EXP(-XSQU)
          ELSE
               ZPDF=0
          ENDIF
          CDF=ZPDF*(A1*T+A2*T**2+A3*T**3)
     END IF
     FPO=1-CDF
     RETURN
     END
***********************************************************************
*                                                                     *
*    GIVEN ITOT AND ISUM ,      THEN  CALCULATE MINIMUM REPLICATE.    *
*                                                                     *
***********************************************************************
      FUNCTION MINIR(ITOT,ISUM)
      INTEGER IR, ITOT,ISUM,ICHECK,MINIR
      IR=2
 10   ICHECK=IR*ITOT-ISUM-1
      IF(ICHECK.LE.0) THEN
         IR=IR+1
         GO TO 10
       ELSE
         MINIR=IR
       ENDIF
      RETURN
      END
```

APPENDIX G
THE POWER OF T-TEST PROGRAM LIST

```
***********************************************************************
*                                                                     *
*    POWER ALGORITHM ( T TEST )                                       *
*                                                                     *
*  DIRECTED BY : PROFESSOR  DONALD.R. BARR                            *
*                                                                     *
*  WRITTEN BY : HUR, SEONG PIL             JULY 1986                  *
*             DEPARTMENT OF  O.R.        NAVAL POSTGRADUATE SCHOOL    *
*                                                                     *
*                                                                     *
*    PROGRAM IS USED THE PRESENTATION OF POWER ALGORITHM OF T-TEST.   *
*       (1) ONE-SAMPLE CASE TEST.                                     *
*             . ONE-SIDED TEST ( HA : MU>MUO )                        *
*             . ONE-SIDED TEST ( HA : MU<MUO )                        *
*             . TWO-SIDED TEST ( HA : MU MUO )                        *
*       (2) TWO-SAMPLE CASE TEST.                                     *
*             . ONE-SIDED TEST ( HA : MU>MUO )                        *
*             . ONE-SIDED TEST ( HA : MU<MUO )                        *
*             . TWO-SIDED TEST ( HA : MU MUO )                        *
*  NOTE : Subroutine PLOTT is NONIMSL subroutine library.             *
*                                                                     *
***********************************************************************
      REAL NN(200),POWER(200),DF,NONCDF,TCDF1,TCDF2,TINV1
      REAL TINV,NONCEN,TX,ALPHA,CDF,CONST ,MAXMU1,MINMU1,INCMU1
      REAL MAXDEL,MINDEL,INCDEL,POW,POW1,POW2,MUZERO,STDV,MU1
      INTEGER JJ,K,N
      CHARACTER*1 ANS
C______________________________________________________________________
C
C     INPUT USER OPTIONS.
C
C______________________________________________________________________
100   PRINT*,'DO YOU WANT TO TEST ONE SAMPLE T-TEST(Y/N)?'
      READ(5,10) ANS
 10   FORMAT(A1)
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 160
      PRINT*,' SAMPLE SIZE ?'
      READ(5,*) N
      PRINT*,' ALPHA-LEVEL ?'
      READ(5,*) ALPHA
      PRINT*,' MU-ZERO VALUE  ?'
      READ(5,*) MUZERO
      PRINT*,' STANDARD DEVIATION VALUE ?'
      READ(5,*) STDV
C______________________________________________________________________
C
C     ONE-SAMPLE ONE-SIDED TEST, HA : MU > MUO.
C
C______________________________________________________________________
      PRINT*,'DO YOU WANT TO USE ONE-SIDED TEST (Y/N)?'
      READ(5,10) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 30
      PRINT*,'DO YOU WANT TO TEST HO:MU=MUO VS HA:MU>MUO (Y/N)?'
      READ(5,10) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 40
      PRINT*,' MAXIMUM MU1 VALUE ( THE MAXIMUM VALUE OF X-AXIS )?'
      READ (5,*) MAXMU1
      PRINT*,' MINIMUM MU1 VALUE ( THE MINIMUM VALUE OF X-AXIS )?
     & (CONDITION :MU1 MUST BE MORE THAN MU-ZERO )'
      READ (5,*) MINMU1
      PRINT*,' LARGE INCREMENT  N VALUE ?'
      READ (5,*) INCMU1
      N=N-1
C______________________________________________________________________
C
C     GENERATE T-INVERSE VALUE ( CRITICAL REGION VALUE ).
C
C______________________________________________________________________
      TINV=TINVER(N,ALPHA)
      JJ=1
```

```
      DO 50 MU1=MINMU1,MAXMU1,INCMU1
C______________________________________________________________________
C
C     COMPUTE NON-CENTRALITY VALUE.
C
C______________________________________________________________________
      DF=N
      NONCEN=((MU1-MUZERO)*SQRT(DF+1))/STDV
C______________________________________________________________________
C
C     GENERATE CDF OF NONCENTRAL T DISTRIBUTION.
C
C______________________________________________________________________
      TCDF=NONCDF(N,NONCEN,TINV)
      POW=1-TCDF
      NN(JJ)=MU1
      POWER(JJ)=POW
      JJ=JJ+1
50    CONTINUE
      JJ=JJ-1
      PRINT*,'NN       POWER'
      DO 60 I=1,JJ
      WRITE(6,70)  NN(I),POWER(I)
 70   FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
 60   CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,10) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 100
      GO TO 80
C______________________________________________________________________
C
C     ONE-SAMPLE ONE-SIDED TEST, HA : MU < MU0.
C
C______________________________________________________________________
 40   PRINT*,'DO YOU WANT TO TEST H0:MU=MU0 VS HA:MU<MU0 (Y/N)?'
      READ(5,10) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 80
      PRINT*,' MAXIMUM MU1 VALUE ( THE MAXIMUM VALUE OF X-AXIS )?'
      READ (5,*) MAXMU1
      PRINT*,' MINIMUM MU1 VALUE ( THE MINIMUM VALUE OF X-AXIS )?
     & (CONDITION :MU1 MUST BE LESS THAN MU-ZERO ) '
      READ (5,*) MINMU1
      PRINT*,' INCREMENT  MU1 VALUE ?'
      READ (5,*)INCMU1
      N=N-1
      TINV=TINVER(N,ALPHA)
      TINV=-TINV
      JJ=1
      DO 90 MU1=MINMU1,MAXMU1,INCMU1
      DF=N
      NONCEN=((MU1-MUZERO)*SQRT(DF+1))/STDV
      TCDF=NONCDF(N,NONCEN,TINV)
      POW=TCDF
      NN(JJ)=MU1
      POWER(JJ)=POW
      JJ=JJ+1
90    CONTINUE
      JJ=JJ-1
      PRINT*,'NN       POWER'
      DO 110 I=1,JJ
      WRITE(6,120)  NN(I),POWER(I)
 120    FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
 110  CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,10) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 100
      GO TO 80
C______________________________________________________________________
C
C     ONE-SAMPLE TWO-SIDED TEST, HA : MU =/ MU0.
C
C______________________________________________________________________
 30   PRINT*,'DO YOU WANT TO USE TWO-SIDED TEST(Y/N)?'
      READ(5,10) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 80
      PRINT*,'MAXIMUM MU1 VALUE ( THE MAXIMUM VALUE ON X-AXIS )?'
      READ (5,*) MAXMU1
      PRINT*,' MINIMUM MU1 VALUE ( THE MINIMUM VALUE ON X-AXIS )?
     & (CONDITION :MU1 MUST BE LESS THAN MU-ZERO )'
      READ (5,*) MINMU1
      PRINT*,' INCREMENT  MU1 VALUE ?'
      READ (5,*)INCMU1
      N=N-1
```

```
      ALPHA=ALPHA/2.
      TINV=TINVER(N,ALPHA)
      JJ=1
      DO 130 MU1=MINMU1,MAXMU1,INCMU1
      DF=N
      NONCEN=((MU1-MUZERO)*SQRT(DF+1.))/STDV
      TCDF1=NONCDF(N,NONCEN,TINV)
      POW1=1.-TCDF1
      TINV1=-TINV
      TCDF2=NONCDF(N,NONCEN,TINV1)
      POW2=TCDF2
      POW=POW1+POW2
      NN(JJ)=MU1
      POWER(JJ)=POW
      JJ=JJ+1
130   CONTINUE
      JJ=JJ-1
      PRINT*,'NN        POWER'
      DO 140 I=1,JJ
      WRITE(6,150)  NN(I),POWER(I)
150   FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
140   CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,10) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 100
      GO TO 80
C_____________________________________________________________________
C
C     TWO-SAMPLE ONE-SIDED TEST, HA : MUX > MUY.
C
C_____________________________________________________________________
160   PRINT*,'DO YOU WANT TO TEST TWO SAMPLE T-TEST(Y/N)?'
      READ(5,10) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 80
      PRINT*,' SAMPLE SIZE FROM POPULATION 1 ?'
      READ(5,*) N
      PRINT*,' SAMPLE SIZE FROM POPULATION 2 ?'
      READ(5,*) M
      PRINT*,' ALPHA-LEVEL ?'
      READ(5,*) ALPHA
      PRINT*,'DO YOU WANT TO USE ONE-SIDED TEST (Y/N)?'
      READ(5,10) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 170
      PRINT*,'DO YOU WANT TO TEST H0: MUX=MUY VS HA: MUX>MUY (Y/N)?'
      READ(5,10) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 180
      PRINT*,' MAXIMUM NONCENTRALITY VALUE ? '
      READ (5,*) MAXDEL
      PRINT*,' MINIMUM NONCENTRALITY VALUE?'
C    & (CONDITION :MU1 MUST BE MORE THAN MU-ZERO )?'
      READ (5,*) MINDEL
      PRINT*,' INCREMENT  NONCENTRALITY VALUE?'
      READ (5,*)INCDEL
      N1=M+N-2
      TINV=TINVER(N1,ALPHA)
      JJ=1
      DO 190 NONCEN=MINDEL,MAXDEL,INCDEL
      TCDF=NONCDF(N1,NONCEN,TINV)
      POW=1-TCDF
      NN(JJ)=NONCEN
      POWER(JJ)=POW
      JJ=JJ+1
190   CONTINUE
      JJ=JJ-1
      PRINT*,'NN        POWER'
      DO 200 I=1,JJ
      WRITE(6,210)  NN(I),POWER(I)
210   FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
200   CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,10) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 100
      GO TO 80
C_____________________________________________________________________
C
C     TWO-SAMPLE ONE-SIDED TEST, HA : MUX < MUY.
C
C_____________________________________________________________________
180   PRINT*,'DO YOU WANT TO TEST H0:MUX=MUY VS HA:MUX<MUY (Y/N)?'
      READ(5,10) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 80
      PRINT*,' MAXIMUM NONCENTRALITY VALUE ? '
      READ (5,*) MAXDEL
```

```
      PRINT*,' MINIMUM NONCENTRALITY VALUE?'
      REAO (5,*) MINDEL
      PRINT*,' INCREMENT  NONCENTRALITY VALUE?'
      REAO (5,*)INCOEL
      N1=M+N-2
      TINV=TINVER(N1,ALPHA)
      TINV2=-TINV
      JJ=1
      DO 220 NONCEN=MINDEL,MAXDEL,INCDEL
      TCDF=NONCDF(N1,NONCEN,TINV2)
      POW=TCOF
      NN(JJ)=NONCEN
      POWER(JJ)=POW
      JJ=JJ+1
220   CONTINUE
      JJ=JJ-1
      PRINT*,'NN       POWER'
      DO 230 I=1,JJ
      WRITE(6,240)  NN(I),POWER(I)
240   FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
230   CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'DO YOU WANT TO PLOT AGAIN (Y/N)?'
      READ (5,10) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 100
      GO TO 80
C____________________________________________________________________
C
C     TWO-SAMPLE TWO-SIOEO TEST, HA : MUX =/ MUY.
C
C____________________________________________________________________
 170  PRINT*,'DO YOU WANT TO USE TWO-SIOED TEST(Y/N)?'
      READ(5,10) ANS
      IF((ANS.EQ.'N').OR.(ANS.EQ.'N')) GO TO 80
      PRINT*,' MAXIMUM NONCENTRALITY VALUE ? '
      READ (5,*) MAXDEL
      PRINT*,' MINIMUM NONCENTRALITY VALUE?'
      REAO (5,*) MINDEL
      PRINT*,'INCREMENT  NONCENTRALITY VALUE?'
      READ (5,*)INCOEL
      N1=M+N-2
      ALPHA1=ALPHA/2.
      TINV=TINVER(N1,ALPHA1)
      JJ=1
      OO 250 NONCEN=MINOEL,MAXOEL,INCOEL
      TCDF1=NONCDF(N1,NONCEN,TINV)
      POW1=1.-TCDF1
      TINV1=-TINV
      TCDF2=NONCOF(N1,NONCEN,TINV1)
      POW2=TCDF2
      POW=POW1+POW2
      NN(JJ)=NONCEN
      POWER(JJ)=POW
      JJ=JJ+1
250   CONTINUE
      JJ=JJ-1
      PRINT*,'NN       POWER'
      OO 260 I=1,JJ
      WRITE(6,270)  NN(I),POWER(I)
270   FORMAT('0', 'NN=',F8.3,4X,'POWER=',F10.5)
260   CONTINUE
      CALL PLOTT(NN,POWER,JJ,0)
      PRINT*,'OO YOU WANT TO PLOT AGAIN (Y/N)?'
      REAO (5,10) ANS
      IF((ANS.EQ.'Y').OR.(ANS.EQ.'Y')) GO TO 100
      GO TO 80
 80   STOP
      ENO
*********************************************************************
*                                                                   *
*       APPROXIMATION  TO THE T- INVERSE OISTRIBUTION.              *
*       CALCULATE T-INVERSE GIVEN OOF ALPHA.                        *
*       Q( T-ALPHA | OOF )= ALPHA.                                  *
*                                                                   *
*********************************************************************
      FUNCTION TINVER(N,ALPHA)
      REAL ALPHA,NUMER,OENUM,LAM,XP,G1,G2,G3,X,TINVER
      INTEGER N
      DATA C0,C1,C2/2.515517,0.802853,0.010328/
      OATA 01,02,03/1.432788,0.189269,0.001308/
C____________________________________________________________________
C
C     CALCULATE NORMAL QUANTILE.
C
C____________________________________________________________________
```

```
      T=SQRT(ALOG(1./(ALPHA**2.)))
      NUMER=C0+C1*T+C2*T**2
      DENUM=1.+D1*T+D2*T**2.+D3*T**3.
      XP=T-(NUMER/DENUM)
C_______________________________________________________________________
C
C     CALCULATE THE INVERSE T-DISTRIBUTION.
C
C_______________________________________________________________________
      X=XP
      G1=.25*(X**3+X)
      G2=(1./96.)*(5.*X**5+16.*X**3+3.*X)
      G3=(1./384.)*(3.*X**7+19.*X**3+17.*X**3-15.*X)
      G4=(1./92160.)*(79.*X**9+776.*X**7+1482.*X**5-1592.*X**3-945.*X)
      TP=X+(G1/N)+(G2/(N*N))+(G3/(N**3))+(G4/(N**4))
      TINVER=TP
      RETURN
      END
***********************  INFORMATION  **********************************
*                                                                      *
*        THE OBJECTIVE OF THIS PROGRAM IS TO CALCULATE NON-CENTRAL T   *
*        DISTRIBUTION (CDF)                                            *
*        THE CALCULATION METHOD IS NORMAL APPROXIMATION.               *
*                                                                      *
**********************  VARIABLE DECLARATION  *************************

      FUNCTION NONCDF(NF,TLAM,TSTAR)
      INTEGER NF
      REAL TSTAR,TLAM,TX, NONCDF,CDF
      DATA A1,A2,A3,P/.4361836,-.1201676,.9372980,.33267/
         CONST  = 1.0 / SQRT (2.0 * 3.1415927)
C_______________________________________________________________________
C
C     CALCULATE THE NORMAL APPROXIMATION .
C
C_______________________________________________________________________
         C1=(1.-(1./(4.*NF)))*TSTAR-TLAM
         C2=SQRT(1.+(TSTAR**2/(2.*NF)))
         TX=C1/C2
         CDF = 0.0
         IF(TX .EQ.0) THEN
         CDF = 0.5
         ELSE IF(TX.GT.0) THEN
              T=1./(1.+P*TX)
              XSQU=(TX*TX)*.5
              IF(XSQU.LT.174) THEN
                   ZPDF=CONST*EXP(-XSQU)
              ELSE
                   ZPDF=0
              ENDIF
              CDF=1-ZPDF*(A1*T+A2*T**2+A3*T**3)
              ELSE
              TX=-TX
              T=1./(1.+P*TX)
              XSQU=(TX*TX)*.5
              IF(XSQU.LT.174) THEN
                   ZPDF=CONST*EXP(-XSQU)
              ELSE
                   ZPDF=0
              ENDIF
              CDF=ZPDF*(A1*T+A2*T**2+A3*T**3)
         END IF

              NONCDF=CDF
         RETURN
         END
```

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